Math 141
Summer III 2017
Exam 3

Name:

1. $\boxed{\text{10 pts.}}$ Use a comparison test to determine convergence or divergence of the series:

$$\sum_{n=1}^{\infty} \frac{\sin(1/n)}{n^2}$$

2. $\boxed{\text{10 pts.}}$ Use the Ratio Test to determine convergence or divergence of the series:

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

3. $\boxed{\text{10 pts.}}$ Choose an appropriate test to determine convergence or divergence of the series:

$$\sum_{n=0}^{\infty} \frac{(-n)^{n-1}}{n!}$$

4. $\boxed{\text{10 pts.}}$ Choose an appropriate test to determine convergence or divergence of the series:

$$\sum_{n=2}^{\infty} \frac{1}{n^{\ln n}}$$

5. $\boxed{\text{10 pts.}}$ Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

converges absolutely, converges conditionally, or diverges. Substantiate your claim with an appropriate test.

6. $\boxed{\text{10 pts.}}$ Estimate the value of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^5+2}$$

with an absolute error less than 10^{-3}. Do not bother to "crunch the numbers" in your final numerical expression.

7. $\boxed{\text{10 pts.}}$ Approximate $e^{0.11}$ using a 3rd-order Taylor polynomial centered at 0.

8. $\boxed{\text{10 pts. each}}$ Determine the interval of convergence of the power series, making sure to test endpoints.

(a) $\displaystyle\sum \frac{x^n}{\sqrt{n^2+3}}$ (b) $\displaystyle\sum \left(2+\frac{2}{n}\right)^n (x-1)^n$ (c) $\displaystyle\sum (\ln n) x^n$

9. $\boxed{\text{15 pts.}}$ Find a power series representation for the function

$$f(x) = \frac{2x^2}{1+x^3},$$

and determine the interval of convergence.

10. $\boxed{\text{15 pts.}}$ Find a power series representation for the function $f(x) = (1+x)^{1/4}$, and determine the interval of convergence.

11. $\boxed{\text{10 pts.}}$ Use a Taylor series to estimate integral with an absolute error less than 10^{-5}:

$$\int_0^{0.1} \frac{\ln(1+x)}{x}\,dx$$

12. $\boxed{\text{10 pts.}}$ For the parametric equations

$$x = 3\cos\theta, \quad y = 2\sin\theta; \quad 0 \le \theta \le 2\pi,$$

eliminate the parameter to obtain the Cartesian equation for the curve.

13. $\boxed{\text{10 pts.}}$ Find parametric equations for the circle centered at $(-2, 9)$ with radius 4, generated clockwise.

14. $\boxed{\text{10 pts.}}$ Convert the polar equation $r\cos\theta = \sin 2\theta$ to Cartesian coordinates.

15. $\boxed{\text{10 pts.}}$ Find the area of the region outside the circle $r = 1/2$ and inside the circle $r = \cos\theta$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}.$$