MATH 141
SUMMER III 2017 NAME:
EXAM 2

1. [10 pts.] Evaluate

$$\int \sin^3 x\sqrt{\cos x}\,dx$$

2. [10 pts.] Use a trigonometric substitution to evaluate

$$\int \frac{\sqrt{9p^2-4}}{p}\,dp$$

3. [10 pts. each] Use partial fractions to find the indefinite integral.

(a) $\displaystyle\int \frac{12r}{(r-4)^2}\,dr$

(b) $\displaystyle\int \frac{x+1}{x^2(x-2)}\,dx$

4. [10 pts. each] Evaluate the improper integral, or show that it diverges.

(a) $\displaystyle\int_2^\infty \frac{1}{y\ln y}\,dy$

(b) $\displaystyle\int_0^{10} \frac{1}{\sqrt[4]{10-x}}\,dx$

5. [10 pts.] Find the area of the region bounded by the graphs of $y = e^{-ax}$ and $y = e^{-bx}$ for $x \geq 0$, where $a > b > 0$.

6. [5 pts. each] Consider the sequence $(4, -4, 4, -4 \ldots)$.

(a) Find a recurrence relation that generates the sequence.

(b) Find an explicit formula for the nth term of the sequence.

7. [10 pts.] Find the limit of the sequence

$$a_n = \left(\frac{1}{n}\right)^{1/\ln n}$$

or determine that it does not exist.

8. [10 pts.] Find the limit of the sequence (b_n), where

$$b_n = \begin{cases} n/(n+1) & \text{if } n \leq 5000 \\ ne^{-n} & \text{if } n > 5000 \end{cases}$$

9. [10 pts.] Evaluate the geometric series or state that it diverges:

$$\sum_{n=1}^{\infty} 3^{-2n}.$$

10. 10 pts. For the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(n+6)(n+7)},$$

find a formula for the kth term of the sequence of partial sums (s_k), and then evaluate the series.

11. 10 pts. Use the Integral Test to determine whether the series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{e^n}$$

12. 10 pts. Use the Integral Test to determine whether the series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$$

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$