Math 141
Summer 2017
Exam 1

Name:

1. [10 pts.] Given that $f(x) = 2x^3 + x - 12$, find $(f^{-1})'(6)$.

2. [10 pts. each] Find the derivative of each function.

(a) $y = x^2 e^{x^3}$

(b) $f(x) = (\ln \sqrt{x}\,)^2$

(c) $g(x) = 3^{\sin x}$

(d) $r(x) = \ln(\ln x^5)$

(e) $\varphi(z) = \cot^{-1}(1/z)$

(f) $y = \tanh(\ln x)$

3. [10 pts.] Find an equation of the line tangent to $y = x^{\sin x}$ at the point $x = 1$.

4. [10 pts. each] Determine each indefinite integral.

(a) $\displaystyle\int \frac{e^x}{4e^x + 6}\,dx$

(b) $\displaystyle\int \frac{1}{(x \ln x)\ln(\ln x)}\,dx$

(c) $\displaystyle\int \frac{e^{\sin x}}{\sec x}\,dx$

(d) $\displaystyle\int \frac{\sinh t}{1 + \cosh t}\,dt.$

5. [10 pts. each] Evaluate each definite integral.

(a) $\displaystyle\int_1^{2e} \frac{3^{\ln x}}{x}\,dx$ (b) $\displaystyle\int_{-\ln\sqrt{3}}^{0} \frac{e^x}{1 + e^{2x}}\,dx$

6. [10 pts.] Evaluate the limit using L'Hôpital's Rule:

$$\lim_{x \to 0^+} x^{1/\ln x}$$

7. [10 pts.] Evaluate

$$\int_0^3 \frac{2}{x^2 + 2x + 2}\,dx$$

8. [10 pts.] Use integration by parts to determine

$$\int t^2 e^{2t}\,dt$$

9. 10 pts. Use integration by parts to evaluate

$$\int_0^{\pi/2} x\cos 2x\,dx$$

10. 10 pts. Find the arc length of the function

$$f(x) = \int_e^x \sqrt{\ln^2 t - 1}\,dt$$

on $[e, e^5]$.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$