Math 141
Summer 2020 Name:
Exam 4

1. (a) $\boxed{\text{10 pts.}}$ Find the quadratic approximating polynomial for $f(x) = \sqrt{x}$, centered at $a = 4$.
   (b) $\boxed{\text{5 pts.}}$ Use the quadratic approximating polynomial to approximate $\sqrt{3.88}$.

2. $\boxed{\text{10 pts.}}$ Find bounds on the absolute error of the estimation

$$\sqrt{1+t} \approx 1 + \frac{t}{2}$$

on the interval $[-0.12, 0.14]$, using the remainder term.

3. $\boxed{\text{10 pts. each}}$ Determine the interval of convergence of the power series.

(a) $\sum \frac{(-1)^n n^2}{(n+1)!}(x+3)^n$ (b) $\sum \frac{6^n}{\sqrt{n}} x^n$ (c) $\sum \frac{(-1)^n}{n^2 3^n}(x-2)^n$

4. $\boxed{\text{15 pts.}}$ Find a power series representation centered at 0 for the function

$$f(x) = \ln \sqrt{1 - x^2},$$

and determine the interval of convergence of the series.

5. $\boxed{\text{10 pts.}}$ Find the first four nonzero terms of the binomial series centered at 0 for

$$f(x) = (1 + 2x)^{3/4}.$$

6. $\boxed{\text{10 pts.}}$ Use Maclaurin series (see table on other side) to evaluate the limit

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x \arctan x}.$$

7. $\boxed{\text{10 pts.}}$ Approximate the value of the definite integral with an absolute error less than 10^{-4}:

$$\int_0^1 \sin \sqrt{x}\, dx.$$

8. $\boxed{\text{10 pts.}}$ Express the curve given by the parametric equations

$$x(t) = \sec t, \quad y(t) = \tan t, \quad 0 \le t \le \pi/4$$

by an equation in x and y (i.e. a Cartesian equation).

9. $\boxed{\text{10 pts.}}$ Find the slope of the curve given by parametric equations

$$x(t) = 4 \sin 2t, \quad y(t) = 3 \cos 2t$$

at the point corresponding to $t = \pi/6$.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}.$$