MATH 141
SUMMER 2020 NAME:
EXAM 3

1. [10 pts. each] Find the limit of each sequence, or show that the limit does not exist.

   (a) $a_n = \left(\frac{n+3}{5n}\right)\left(2 - \frac{9}{n}\right)$

   (b) $a_n = n - \sqrt{n^2 - n}$

   (c) $a_n = \frac{\ln n}{\ln 2n}$

2. [5 pts. each] Consider the sequence $\{-2, -8, -14, -20, \ldots\}$.

   (a) Find a recurrence relation that generates the sequence.

   (b) Find an explicit formula for the nth term of the sequence.

3. [10 pts.] Evaluate the geometric series or explain why it diverges:

$$\sum_{n=1}^{\infty} 3\left(\frac{1}{2}\right)^{2n}$$

4. [10 pts.] Either show the telescoping series

$$\sum_{n=1}^{\infty} \frac{6}{n^2 + 2n}$$

   diverges, or evaluate the series.

5. [10 pts.] Write the repeating decimal $2.0\overline{45}$ as a geometric series and then as a fraction (i.e. a ratio of integers).

6. [10 pts. each] Determine whether the series converges or diverges, using an appropriate test and justifying all work. Arguments must be clear and thorough.

   (a) $\sum_{n=3}^{\infty} \frac{4}{n\sqrt{\ln n}}$

   (b) $\sum_{n=1}^{\infty} \frac{3^n}{n^2 + 1}$

   (c) $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$

   (d) $\sum_{k=1}^{\infty} \frac{2 + \sin k}{k}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{e^n - 1}$

(f) $\displaystyle\sum_{j=1}^{\infty} \frac{4^{j^2}}{j!}$

7. 10 pts. Find the values of the parameter $p > 0$ for which the series converges:

$$\sum_{n=2}^{\infty} \frac{10}{(\ln n)^p}$$

8. 10 pts. each Use the Alternating Series Test to show the series converges, or use some other test to show it diverges. If the series converges, use any test to determine whether it converges absolutely or conditionally.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{0.99}}$

(b) $\displaystyle\sum_{m=1}^{\infty} (-1)^m \frac{m^2+1}{3m^4+3}$

Some Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$