MATH 141
SUMMER 2020
EXAM 2

NAME:

1. [10 pts. each] Evaluate the integral.

(a) $\displaystyle\int \frac{3x}{\sqrt{x+6}}\,dx$

(b) $\displaystyle\int_{\sqrt{2}}^{2} \frac{\sqrt{x^2-1}}{x}\,dx$

(c) $\displaystyle\int \frac{2x^2+7x+4}{x^3+2x^2+2x}\,dx$

(d) $\displaystyle\int \frac{\sin^3 t}{\cos^5 t}\,dt$

(e) $\displaystyle\int x\tan^{-1} 7x\,dx$

(f) $\displaystyle\int_1^e x^2 \ln x\,dx$

(g) $\displaystyle\int \frac{dx}{\sqrt{16+4x^2}}$

(h) $\displaystyle\int \frac{x^4+1}{x^3+9x}\,dx$

2. [10 pts.] Find the volume of the solid generated by revolving the region bounded by $y = \dfrac{1}{\sqrt{x(3-x)}}$, $y = 0$, $x = 1$, and $x = 2$ about the x-axis.

3. [10 pts. each] Determine whether the improper integral is convergent or divergent, and evaluate if convergent.

(a) $\displaystyle\int_1^\infty \frac{dv}{v(v+1)}$

(b) $\displaystyle\int_{-3}^{1} \frac{dy}{(2y+6)^{2/3}}$

(c) $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{\sqrt{e^{2x}-1}}\,dx$

4. [10 pts.] Let $\mathcal{R}$ be the region bounded by the graphs of $y = e^{-ax}$ and $y = e^{-bx}$ for $x \geq 0$, where $a > b > 0$. Find the area of R in terms of a and b.

5. [10 pts.] Use the Comparison Theorem to determine whether the integral converges or diverges:

$$\int_1^\infty \frac{2+\cos x}{\sqrt{x}}\,dx.$$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$