MATH 141
SUMMER 2019 NAME:
EXAM 3

1. [10 pts. each] Find the limit of each sequence (a_n), or show that the limit does not exist.

   (a) $a_n = 2 + (0.98)^n$

   (b) $a_n = \ln(3n^2 + 4) - \ln(n^2 + 4)$

2. [10 pts.] Determine whether the sequence $a_n = 3 - 2ne^{-n}$ is increasing, decreasing, or not monotonic. Is the sequence bounded?

3. [10 pts.] Determine whether the geometric series

$$10 - 2 + 0.4 - 0.08 + \cdots$$

is convergent or divergent. If convergent, find the sum.

4. [10 pts.] Determine whether the telescoping series

$$\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$$

converges or diverges. If it converges, find the sum.

5. [10 pts.] Find the value of c if

$$\sum_{n=1}^{\infty} (c+3)^{-n} = 4.$$

6. [10 pts.] Use the Integral Test to determine whether the series converges or diverges:

$$\frac{1}{3} + \frac{1}{7} + \frac{1}{11} + \frac{1}{15} + \frac{1}{19} + \cdots.$$

7. [10 pts. each] Use either comparison test to determine whether the series converges or diverges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n \sin^2 n}{n^3 + 1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n - 2}$

8. [10 pts.] By definition the decimal representation $0.d_1d_2d_3\ldots$ denotes a series,

$$0.d_1d_2d_3\ldots = \frac{d_1}{10} + \frac{d_2}{10^2} + \frac{d_3}{10^3} + \cdots,$$

where of course each d_k is some integer from 0 to 9. Show that the series always converges.

9. [10 pts. each] Use the Alternating Series Test or Divergence Test to determine whether the series converges or diverges, respectively.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n^2}{n^2+n+1}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{2n+3}$

10. [10 pts. each] Use the Ratio Test or Root Test to determine whether the series converges or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n+1)!}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(\ln n)^n}$

11. [10 pts. each] Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^2}$

Some Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|+c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x|+c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x|+c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x+\tan x|+c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x+\cot x|+c$