MATH 141
SUMMER 2018 NAME:
EXAM 2

1. $\boxed{\text{10 pts.}}$ Find the integral

$$\int \frac{8}{t^{-2}+1}\,dt$$

2. $\boxed{\text{10 pts. each}}$ Evaluate each of the following.

(a) $\displaystyle\int_0^{\pi/4} x^2 \sin 2x\,dx$

(b) $\displaystyle\int (\ln x)^2\,dx$

3. $\boxed{\text{10 pts.}}$ Show that if f has a continuous second derivative on $[a, b]$ and $f'(a) = f'(b) = 0$, then

$$\int_a^b x f''(x)\,dx = f(a) - f(b).$$

4. $\boxed{\text{10 pts. each}}$ Find each indefinite trigonometric integral.

(a) $\displaystyle\int \sin^5 x \cos^{-2} x\,dx$

(b) $\displaystyle\int \cot^4 x\,dx$

(c) $\displaystyle\int \tan^9 x \sec^4 x\,dx$

5. $\boxed{\text{10 pts. each}}$ Use a trigonometric substitution to find the indefinite integral

(a) $\displaystyle\int_{1/2}^{1} \frac{\sqrt{1-y^2}}{y^2}\,dy$

(b) $\displaystyle\int \frac{1}{x^2\sqrt{9x^2-1}}\,dx, \quad x > \frac{1}{3}$

6. $\boxed{\text{10 pts. each}}$ Use partial fractions to find the indefinite integral

(a) $\displaystyle\int \frac{8}{(y-4)^2(y+3)}\,dy$

(b) $\displaystyle\int \frac{2}{(x-4)(x^2+2x+6)}\,dx$

7. $\boxed{\text{10 pts.}}$ Evaluate the improper integral or state that it diverges:

$$\int_0^{\infty} e^{-ax}\,dx, \quad a > 0.$$

8. $\boxed{\text{10 pts.}}$ Evaluate the improper integral, or state that it diverges:

$$\int_0^1 \frac{t^3}{t^4 - 1}\,dt.$$

9. $\boxed{\text{10 pts.}}$ Let $\mathcal{R}$ be the region on the xy-plane bounded by

$$f(x) = \frac{1}{\sqrt{x}\ln x}$$

and the x-axis on the interval $[2, \infty)$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1 - x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1 + x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2 - 1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$