MATH 141
SUMMER 2018
EXAM 1

NAME:

1. Consider the function $f(x) = 3x + \sin x$.

   (a) $\boxed{\text{5 pts.}}$ Show that f is one-to-one and hence has an inverse.

   (b) $\boxed{\text{10 pts.}}$ Find $(f^{-1})'(0)$ using the Inverse Function Theorem

2. $\boxed{\text{10 pts. each}}$ Find the derivative of each function.

   (a) $y = \dfrac{\ln x}{\ln x + 1}$

   (b) $f(x) = \sin(\cos(e^x))$

   (c) $g(x) = (\cot x)^{x-1}$

   (d) $h(t) = t^{(t^{1/2})}$

   (e) $\ell(x) = 4\log_3(1 - x^3)$

   (f) $\varphi(u) = \tan^{-1}(4u - 4)$

   (g) $y = x^2 \cosh^3 5x$

3. $\boxed{\text{10 pts.}}$ Find an equation of the line tangent to $y = 2e^x + 8$ at the point where $x = \ln 6$.

4. $\boxed{\text{10 pts. each}}$ Determine each indefinite integral.

   (a) $\displaystyle\int x^5 e^{x^6}\,dx$

   (b) $\displaystyle\int \frac{1}{(x\ln x)\ln(\ln x)}\,dx$

   (c) $\displaystyle\int \frac{e^{\cos x}}{\csc x}\,dx$

5. $\boxed{\text{10 pts. each}}$ Evaluate each definite integral.

   (a) $\displaystyle\int_{-2}^{2} \frac{e^{t/2}}{e^{t/2}+1}\,dt$

   (b) $\displaystyle\int_{0}^{3/2} \frac{1}{\sqrt{36-4x^2}}\,dx$

   (c) $\displaystyle\int_{0}^{1} \cosh^3 4x \sinh 4x\,dx$

6. $\boxed{\text{10 pts.}}$ Evaluate the limit using L'Hôpital's Rule: $\displaystyle\lim_{x\to\infty}(2x^3+1)^{1/\ln x}$.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$