Math 141
Summer 2017 Name:
Exam 3

1. $\boxed{\text{5 pts. each}}$ Consider the sequence $(4, -4, 4, -4 \ldots)$.

   (a) Find a recurrence relation that generates the sequence.

   (b) Find an explicit formula for the nth term of the sequence.

2. $\boxed{\text{10 pts.}}$ Find the limit of the sequence

$$a_n = \left(\frac{1}{n}\right)^{1/\ln n}$$

or determine that it does not exist.

3. $\boxed{\text{10 pts.}}$ Find the limit of the sequence (b_n), where

$$b_n = \begin{cases} n/(n+1) & \text{if } n \leq 5000 \\ ne^{-n} & \text{if } n > 5000 \end{cases}$$

4. $\boxed{\text{10 pts.}}$ Evaluate the geometric series or state that it diverges:

$$\sum_{n=1}^{\infty} 3^{-2n}.$$

5. $\boxed{\text{10 pts.}}$ For the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(n+6)(n+7)},$$

find a formula for the kth term of the sequence of partial sums (s_k), and then evaluate the series.

6. $\boxed{\text{10 pts.}}$ Use the Integral Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{1}{e^n}$$

converges or diverges, or state that the test does not apply.

7. $\boxed{\text{10 pts.}}$ Use either the Direct Comparison Test or Limit Comparison Test to determine whether

$$\sum_{n=1}^{\infty} \frac{1}{2n - \sqrt[3]{n^2}}$$

converges or diverges.

8. $\boxed{\text{10 pts.}}$ Use the Ratio Test to determine whether

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$$

converges or diverges.

9. $\boxed{\text{10 pts.}}$ Choose an appropriate test to determine whether

$$\sum_{n=0}^{\infty} \frac{(-n)^{n-1}}{n!}$$

converges or diverges.

10. $\boxed{\text{10 pts.}}$ Choose an appropriate test to determine whether the series

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \cdots$$

converges or diverges.

11. $\boxed{\text{10 pts.}}$ Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

converges absolutely, converges conditionally, or diverges. Substantiate your claim with an appropriate test.

12. $\boxed{\text{10 pts.}}$ Estimate the value of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^4 + 1}$$

with an absolute error less than 10^{-4}. Do not bother to "crunch the numbers" in your final numerical expression.