MATH 141
SUMMER 2017 NAME:
EXAM 2

1. [10 pts.] Evaluate
$$\int_1^3 \frac{2}{x^2+2x+1}\,dx$$

2. [10 pts.] Use integration by parts to determine
$$\int t^2 e^{2t}\,dt$$

3. [10 pts.] Use integration by parts to evaluate
$$\int_0^{\pi/2} x\cos 2x\,dx$$

4. [10 pts.] Find the arc length of the function
$$f(x)=\int_e^x \sqrt{\ln^2 t-1}\,dt$$
on $[e, e^4]$.

5. [10 pts.] Use a trigonometric substitution to evaluate
$$\int_6^{6\sqrt{3}} \frac{q^2}{(q^2+36)^2}dq.$$

6. [10 pts. each] Use partial fractions to find the indefinite integral.
   (a) $\displaystyle\int \frac{12r}{(r-4)^2}\,dr$
   (b) $\displaystyle\int \frac{x+1}{x^2(x-2)}\,dx$

7. [10 pts. each] Evaluate the improper integral, or show that it diverges.
   (a) $\displaystyle\int_2^\infty \frac{1}{y\ln y}\,dy$
   (b) $\displaystyle\int_0^{10} \frac{1}{\sqrt[4]{10-x}}\,dx$

8. [10 pts.] Find the area of the region bounded by the graphs of $y=e^{-ax}$ and $y=e^{-bx}$ for $x\geq 0$, where $a>b>0$.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\Big(\frac{x}{a}\Big) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\Big(\frac{x}{a}\Big) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\Big|\frac{x}{a}\Big| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$