MATH 141
SUMMER 2017 NAME:
EXAM 1

1. [10 pts.] Given that $f(x) = 2x^3 + x - 12$, find $(f^{-1})'(6)$.

2. [10 pts. each] Find the derivative of each function.
   (a) $f(x) = \sqrt{\ln \sqrt{x}}$
   (b) $g(x) = x^{\ln x}$
   (c) $h(t) = (\tan t)^{t^2}$
   (d) $r(x) = \log_7 \sqrt[3]{8x}$
   (e) $\varphi(z) = \cot^{-1}(1/z)$
   (f) $y = \sinh^3(e^{-4x})$

3. [10 pts.] Find an equation of the line tangent to $y = x^{\sin x}$ at the point $x = 1$.

4. [10 pts. each] Determine each indefinite integral.
   (a) $\displaystyle\int \frac{e^x}{4e^x + 6}\,dx$
   (b) $\displaystyle\int \frac{1}{(x \ln x)\ln(\ln x)}\,dx$
   (c) $\displaystyle\int \frac{e^{\sin x}}{\sec x}\,dx$
   (d) $\displaystyle\int \frac{\sinh t}{1 + \cosh t}\,dt.$

5. [10 pts. each] Evaluate each definite integral.
   (a) $\displaystyle\int_1^{2e} \frac{3^{\ln x}}{x}\,dx$ (b) $\displaystyle\int_{-\ln\sqrt{3}}^{0} \frac{e^x}{1 + e^{2x}}\,dx$

6. [10 pts.] Evaluate the limit using L'Hôpital's Rule: $\displaystyle\lim_{x \to -5^-} x^{4/(x+5)}$

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$