MATH 141
SUMMER 2016
EXAM 2

NAME:

1. $\boxed{\text{10 pts.}}$ Use a substitution to find the integral: $\displaystyle\int \frac{e^x}{e^x - 2e^{-x}}\,dx$.

2. $\boxed{\text{10 pts. each}}$ Use integration by parts to determine each of the following.

   (a) $\displaystyle\int \frac{\ln t}{t^{10}}\,dt$

   (b) $\displaystyle\int x^2 e^{-3x}\,dx$

3. $\boxed{\text{10 pts. each}}$ Find each indefinite trigonometric integral.

   (a) $\displaystyle\int \sin^3\theta\cos^{-2}\theta\,d\theta$

   (b) $\displaystyle\int \tan^5 x\sec^3 x\,dx$

4. $\boxed{\text{10 pts. each}}$ Use a trigonometric substitution to find the indefinite integral

   (a) $\displaystyle\int_{1/2}^{1} \frac{\sqrt{s^2+1}}{s^2}\,ds$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{16x^2-1}}\,dx, \quad x < -\frac{1}{4}$

5. $\boxed{\text{10 pts. each}}$ Use partial fractions to find the indefinite integral.

   (a) $\displaystyle\int \frac{12}{(r-4)(r+3)}\,dr$

   (b) $\displaystyle\int \frac{z+1}{z(z^2+4)}\,dz$

6. $\boxed{\text{10 pts. each}}$ Evaluate the improper integral, or show that it diverges.

   (a) $\displaystyle\int_{-\infty}^{-2} \frac{2}{t^2-1}\,dt$

   (b) $\displaystyle\int_{0}^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

Formulas

- $\tan x = y \;\Leftrightarrow\; \tan^{-1} y = x, \quad \text{for } x \in (-\pi/2, \pi/2)$
- $\cot x = y \;\Leftrightarrow\; \cot^{-1} y = x, \quad \text{for } x \in (0, \pi)$
- $\sec x = y \;\Leftrightarrow\; \sec^{-1} y = x, \quad \text{for } x \in [0, \pi/2) \cup (\pi/2, \pi]$
- $\csc x = y \;\Leftrightarrow\; \csc^{-1} y = x, \quad \text{for } x \in [-\pi/2, 0) \cup (0, \pi/2]$
- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + c$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c, \quad \int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c, \quad \int \csc x\,dx = -\ln|\csc x + \cot x| + c$