Math 141
Summer 2014
Exam 4

Name:

1. $\boxed{\text{15 pts. each}}$ Determine the interval of convergence and radius of convergence of the power series, making sure to test endpoints.

(a) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{3x-2}{5}\right)^k$ (b) $\displaystyle\sum_{k=1}^{\infty}\frac{(x+4)^k}{3k}$ (c) $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{(x-5)^k}{k\cdot 2^k}$

2. $\boxed{\text{10 pts.}}$ Use the geometric series to find the power series representation (centered at 0) of

$$g(x)=\frac{8}{2x+1}.$$

Give the interval of convergence of the new series.

3. $\boxed{\text{15 pts.}}$ Find the function represented by the series

$$\sum_{k=0}^{\infty}(\sqrt{x}+4)^k,$$

and give the interval of convergence.

4. Let $f(x)=\ln(x)$.

(a) $\boxed{\text{15 pts.}}$ Find the first four nonzero terms of the Taylor series for f centered at 3.

(b) $\boxed{\text{5 pts.}}$ Write the Taylor series using summation notation.

5. $\boxed{\text{15 pts.}}$ Use a Taylor series to approximate the value of the definite integral

$$\int_0^{0.3}\cos(x^3)\,dx$$

with an absolute error less than 10^{-10}.

6. $\boxed{\text{10 pts.}}$ Consider the parametric equations

$$x=\sqrt[3]{t}+4,\quad y=5t-3;\quad 0\le t\le 27.$$

Eliminate the parameter to obtain an equation of the form $y=f(x)$. What is the domain of f?

7. $\boxed{\text{10 pts.}}$ Express the Cartesian coordinates $(-1,-\sqrt{3})$ in polar coordinates in three different ways.

8. $\boxed{\text{10 pts.}}$ Find the slope of the tangent line to the polar curve $r=8\cos\theta$ at the point $(4,5\pi/6)$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \text{ for } |x| < 1 \text{ (Geometric Series)}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \text{ for } |x| \leq 1$$

Some Trigonometric Identities:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$