MATH 141
SUMMER 2014 NAME:
EXAM 3

1. $\boxed{\text{10 pts. each}}$ The first six terms of the sequence $(a_n)_{n=1}^{\infty}$ are $4, 7, 10, 13, 16, 19, \ldots$

   (a) Assuming the pattern established by the first six terms continues indefinitely, find a recurrence relation that generates $(a_n)_{n=1}^{\infty}$.

   (b) Find an explicit formula for the nth term of the sequence.

2. $\boxed{\text{10 pts.}}$ A radioactive isotope transmutes 30% of its mass to another element every 10 years. Let M_n be the mass of the isotope at the end of the nth decade, where the initial mass of the material is $M_0 = 50$ grams. Find an explicit formula for M_n, the nth term of the sequence $(M_n)_{n=0}^{\infty}$.

3. $\boxed{\text{10 pts.}}$ Find the limit of the sequence

$$\left(\frac{4n - 5n^6}{11n^6 - 9}\right)_{n=1}^{\infty}$$

   or determine that the limit does not exist.

4. $\boxed{\text{10 pts.}}$ Evaluate the geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{6}\right)^n 3^{4-n},$$

   if it converges.

5. $\boxed{\text{10 pts.}}$ For the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{(n+2)(n+4)},$$

   find a formula for the nth term of the sequence of partial sums $(s_n)_{n=1}^{\infty}$, then evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

6. $\boxed{\text{10 pts. each}}$ Determine whether the series converges or diverges using the indicated test, or state that the test is inconclusive.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$, using the Integral Test.

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n+1000}$, using the Divergence Test.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$, using the Ratio Test.

(d) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{2n^2}$, Root Test.

(e) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2+8}$, using the Comparison Test or Limit Comparison Test.

(f) $\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^2+1}}{\sqrt{n^3+2}}$, using the Comparison Test or Limit Comparison Test.

7. $\boxed{\text{10 pts. each}}$ Use the Alternating Series Test to show the series converges, or use another test to show it diverges.

(a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{n}{\ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{n+6}$