

1. 10 pts. each Use integration by parts (possibly more than once) to find each integral.

(a) $\int_{1/2}^{\sqrt{3}/2} \sin^{-1} z \, dz$

(b) $\int x^2 \ln^2 x \, dx$

2. 10 pts. each Find each indefinite trigonometric integral.

(a) $\int \sin^7(x) \cos^3(x) \, dx$

(b) $\int \frac{\csc^4 t}{\cot^2 t} \, dt$

(c) $\int_0^{\pi/2} \sqrt{1 - \cos 2x} \, dx$

3. 10 pts. each Use a trigonometric substitution to find the indefinite integral

(a) $\int_0^{1/3} \frac{1}{(9x^2 + 1)^{3/2}} \, dx$

(b) $\int \sqrt{169 - t^2} \, dt$

4. 10 pts. each Use partial fractions to find the indefinite integral

(a) $\int \frac{2}{x^3 + x^2} \, dx$

(b) $\int \frac{2}{(x - 4)(x^2 + 2x + 6)} \, dx$

5. 10 pts. Evaluate the improper integral

$$\int_{-\infty}^1 \frac{1}{(2x - 3)^3} \, dx$$

or show that it diverges using an appropriate test.

6. 10 pts. Find the area of the region under the curve

$$y = \frac{2}{4x^2 - 1}$$

to the right of $x = 1$. (Hint: use partial fractions.)

FORMULAS & DEFINITIONS

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan \theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot \theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec \theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc \theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
6. $(\tan^{-1} x)' = \frac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
7. $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, -1) \cup (1, \infty)$
8. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c$, for $a \in (0, \infty)$
9. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$, for $a \neq 0$
10. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + c$, for $a \in (0, \infty)$
11. $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
12. $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
13. $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$, $n \neq 1$
14. $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$, $n \neq 1$
15. $\int \tan x dx = -\ln |\cos x| + c = \ln |\sec x| + c$
16. $\int \cot x dx = \ln |\sin x| + c$
17. $\int \sec x dx = \ln |\sec x + \tan x| + c$
18. $\int \csc x dx = -\ln |\csc x + \cot x| + c$