Math 141
Summer 2013
Exam 4

Name:

1. [15 pts. each] Determine the interval of convergence and radius of convergence of the power series, making sure to test endpoints.

(a) $\displaystyle\sum_{k=0}^{\infty}\left(\frac{x+1}{8}\right)^k$ (b) $\displaystyle\sum_{k=1}^{\infty}\frac{(2x+3)^k}{6k}$ (c) $\displaystyle\sum_{k=1}^{\infty}(-1)^k\frac{(x+2)^k}{k\cdot 2^k}$

2. [10 pts.] Use the geometric series to find the power series representation (centered at 0) of

$$h(x)=\frac{2}{3x+1}.$$

Give the interval of convergence of the new series.

3. [15 pts.] Find the function represented by the series

$$\sum_{k=0}^{\infty}(\sqrt{x}+4)^k,$$

and give the interval of convergence.

4. Let $f(x)=\sin(3x)$.

(a) [10 pts.] Find the first four nonzero terms of the Maclaurin series for f.

(b) [5 pts.] Write the power series using summation notation.

(c) [10 pts.] Determine the interval of convergence for the series.

5. [15 pts.] Use a Taylor series to approximate the value of the definite integral

$$\int_0^{0.2}\sin(x^2)\,dx$$

with an absolute error less than 10^{-10}.

6. [10 pts.] Consider the parametric equations

$$x=\sqrt{t}+4,\quad y=3\sqrt{t};\quad 0\le t\le 16.$$

Eliminate the parameter to obtain an equation in x and y.

7. [10 pts.] Express the Cartesian coordinates $(-1,\sqrt{3})$ in polar coordinates in three different ways.

8. [10 pts.] Find the slope of the tangent line to the polar curve $r=8\sin\theta$ at the point $(4,5\pi/6)$.

9. [10 pts.] Find all points where the polar curve $r=3+5\sin\theta$ has a horizontal tangent line.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \text{ for } |x| < 1 \text{ (Geometric Series)}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \text{ for } |x| \leq 1$$

Some Trigonometric Identities:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$