Math 141
Summer 2012
Exam 3

Name:

1. $\boxed{\text{10 pts. each}}$ Use the Alternating Series Test to show the series converges, or use another test to show it diverges.

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k \ln^2 k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \left(1 - \frac{2}{k}\right)$

2. $\boxed{\text{15 pts. each}}$ Determine the interval of convergence and radius of convergence of the power series, making sure to test endpoints.

   (a) $\displaystyle\sum_{k=0}^{\infty} \left(\frac{x+1}{8}\right)^k$

   (b) $\displaystyle\sum_{k=1}^{\infty} \frac{(2x+3)^k}{6k}$

3. $\boxed{\text{10 pts.}}$ Use the geometric series to find the power series representation (centered at 0) of

$$g(x) = \frac{5}{1-6x}.$$

   Give the interval of convergence of the new series.

4. $\boxed{\text{10 pts.}}$ Find the function represented by the series

$$\sum_{k=0}^{\infty} (\sqrt{x} - 7)^k,$$

   and give the interval of convergence.

5. $\boxed{\text{10 pts.}}$ Approximate the value of the convergent series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3}$$

   with an absolute error less than 10^{-4}.

6. Let $f(x) = \cos(4x)$.

   (a) $\boxed{\text{10 pts.}}$ Find the first four nonzero terms of the Maclaurin series for f.

   (b) $\boxed{\text{5 pts.}}$ Write the power series using summation notation.

   (c) $\boxed{\text{10 pts.}}$ Determine the interval of convergence for the series.

7. $\boxed{\text{10 pts.}}$ Evaluate

$$\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x}$$

   using Taylor series. (Do *not* use L'Hôpital's Rule.)

8. $\boxed{\text{10 pts.}}$ Use a Taylor series to approximate

$$\int_0^{0.15} \frac{\sin x}{x} dx,$$

   retaining as many terms as needed to ensure the error is less than 10^{-4}.

9. $\boxed{\text{10 pts.}}$ Consider the parametric equations

$$x = \sqrt{t} + 4, \quad y = 3\sqrt{t}; \quad 0 \le t \le 16.$$

   Eliminate the parameter to obtain an equation in x and y.

10. $\boxed{\text{10 pts.}}$ Express the Cartesian coordinates $(-1, \sqrt{3})$ in polar coordinates in three different ways.

11. $\boxed{\text{10 pts.}}$ Find the slope of the tangent line to the polar curve $r = 8 \sin\theta$ at the point $(4, 5\pi/6)$.

12. $\boxed{\text{10 pts.}}$ Find all points where the polar curve $r = 3 + 5\sin\theta$ has a horizontal tangent line.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \text{ for } |x| < 1 \text{ (Geometric Series)}$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \text{ for } |x| \leq 1$$

Some Trigonometric Identities:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$