MATH 141
SUMMER 2012
EXAM 2

NAME:

1. [10 pts. each] Use a trigonometric substitution to determine the indefinite integral

   (a) $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx,\ x>3$

   (b) $\displaystyle\int \sqrt{121-x^2}\,dx$

2. [10 pts. each] Use partial fractions to determine the indefinite integral

   (a) $\displaystyle\int \frac{3}{x^3-x^2-12x}\,dx$

   (b) $\displaystyle\int \frac{2}{(x-4)(x^2+2x+6)}\,dx$

3. [10 pts. each] Evaluate the improper integral or state that it diverges.

   (a) $\displaystyle\int_{-\infty}^{1} \frac{1}{(2x-3)^2}\,dx$

   (b) $\displaystyle\int_{0}^{4} \frac{1}{x-1}\,dx$

4. [10 pts. each] Find the limit of the sequence, or explain why the limit does not exist.

   (a) $\left\{\dfrac{5n^8}{\sqrt{36n^{16}-10n^{10}}}\right\}$

   (b) $a_n=(-1)^n\sqrt[n]{n}$

5. [10 pts.] Evaluate the geometric series, if it converges: $\displaystyle\sum_{k=2}^{\infty}\frac{3}{(-2)^k}$.

6. [10 pts.] For the telescoping series

$$\sum_{k=1}^{\infty}\frac{1}{(k+1)(k+2)},$$

   find a formula for the nth term of the sequence of partial sums $\{s_n\}$, then evaluate $\lim\limits_{n\to\infty} s_n$ to obtain the value of the series.

7. [10 pts. each] Determine whether the series converges or diverges using one of the indicated tests.

   (a) $\displaystyle\sum_{k=1}^{\infty}\frac{k}{\sqrt{k^2+25}}$, Divergence or Integral Test

   (b) $\displaystyle\sum_{k=1}^{\infty} ke^{-2k^2}$, Divergence or Integral Test

   (c) $\displaystyle\sum_{k=1}^{\infty}\frac{(k!)^2}{(2k)!}$, Ratio Test

   (d) $\displaystyle\sum_{k=1}^{\infty}\frac{k^2}{2^k}$, Root Test

   (e) $\displaystyle\sum_{k=1}^{\infty}\frac{\sin^2 k}{k\sqrt{k}}$, either comparison test

   (f) $\displaystyle\sum_{k=1}^{\infty}\frac{k^7}{k^9+3}$, either comparison test

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(b^x)' = b^x \ln b$, for $b \in (0,1) \cup (1, \infty)$
6. $(\log_b |x|)' = \dfrac{1}{x \ln b}$, for $x \neq 0$
7. $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
8. $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
9. $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
10. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
11. $\displaystyle\int b^x\,dx = \frac{1}{\ln b} b^x + C$, for $b \in (0,1) \cup (1, \infty)$
12. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$, for $a \in (0, \infty)$
13. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$, for $a \neq 0$
14. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$, for $a \in (0, \infty)$
15. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
16. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
17. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, \;\; n \neq 1$
18. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, \;\; n \neq 1$
19. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$
20. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$
21. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$