MATH 141
SUMMER 2012
EXAM 1

NAME:

1. [10 pts.] Given that $f(x) = 2x^3 + x - 12$, find $(f^{-1})'(6)$.

2. [10 pts.] Find all the inverses associated with $f(x) = (x-4)^2$, and state their domains.

3. [10 pts. each] Find the derivative of each function.
   (a) $f(x) = \ln(e^{2x} + 3)$
   (b) $g(x) = x^{\ln(x^3)}$
   (c) $h(x) = (\tan x)^{\cos x}$
   (d) $k(x) = 7\log_3(4 - x^5)$
   (e) $\ell(x) = \sin^{-1}(e^{-2x})$
   (f) $p(x) = \cot^{-1}(\sqrt{x})$

4. [10 pts. each] Determine each indefinite integral.
   (a) $\displaystyle\int (3e^{-8x} - 8e^{11x})\,dx$
   (b) $\displaystyle\int \frac{4}{3-10x}\,dx$
   (c) $\displaystyle\int x^3 9^{x^4}\,dx$

5. [10 pts. each] Evaluate each definite integral.
   (a) $\displaystyle\int_1^{3e} \frac{e^{\ln(x)}}{2x}\,dx$
   (b) $\displaystyle\int_2^{2\sqrt{3}} \frac{5}{z^2+4}\,dz$

6. [15 pts.] Evaluate the limit using L'Hôpital's Rule:
$$\lim_{x\to\infty}\left(\frac{1}{5x}\right)^{2/x}$$

7. [10 pts. each] Use integration by parts to find each integral.
   (a) $\displaystyle\int \theta\sec^2(\theta)\,d\theta$
   (b) $\displaystyle\int e^{-x}\sin(3x)\,dx$
   (c) $\displaystyle\int_{1/2}^{\sqrt{3}/2} \sin^{-1}(y)\,dy$

8. [10 pts. each] Determine each indefinite trigonometric integral.
   (a) $\displaystyle\int \sin^7(x)\cos^3(x)\,dx$
   (b) $\displaystyle\int \tan^3(6x)\,dx$
   (c) $\displaystyle\int_{-\pi/3}^{\pi/3} \sqrt{\sec^2(\varphi) - 1}\,d\varphi$

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(b^x)' = b^x \ln b$, for $b \in (0,1) \cup (1, \infty)$
6. $(\log_b |x|)' = \dfrac{1}{x \ln b}$, for $x \neq 0$
7. $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
8. $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
9. $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
10. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$
11. $\displaystyle\int b^x\,dx = \frac{1}{\ln b} b^x + C$, for $b \in (0,1) \cup (1, \infty)$
12. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$, for $a \in (0, \infty)$
13. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$, for $a \neq 0$
14. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C$, for $a \in (0, \infty)$
15. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
16. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
17. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, \;\; n \neq 1$
18. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, \;\; n \neq 1$
19. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$
20. $\displaystyle\int \cot x\,dx = \ln|\sin x| + C$
21. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$