Math 141
Exam #3
Summer 2011

Name:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, for $|x| < \infty$

$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}$, for $-1 < x \leq 1$

$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$, for $|x| < \infty$

$\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$, for $|x| \leq 1$

Remainder Theorem: Let $R_n = |S - S_n|$ be the remainder in approximating the value of a convergent alternating series $\sum_{k=1}^{\infty}(-1)^{k+1}a_k$ by the sum of its first n terms. Then $R_n \leq a_{n+1}$.

1. $\boxed{\text{10 pts. each}}$ If a series converges, use the Alternating Series Test to show it; otherwise, use some other test to show divergence.

   (a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k^2+4}}$

   (b) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2k^2+3}{5k^2+1}$

2. $\boxed{\text{10 pts.}}$ Estimate the value of the convergent series $\sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)^3}$ with an absolute error less than 10^{-3}.

3. $\boxed{\text{15 pts. each}}$ Determine the interval of convergence and radius of convergence of the power series, making sure to test enpoints.

   (a) $\sum_{k=0}^{\infty} \left(\frac{x-1}{5}\right)^k$

   (b) $\sum_{k=1}^{\infty} \frac{(2x+3)^k}{6k}$

4. $\boxed{\text{10 pts.}}$ Use the geometric series

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \quad |x| < 1$$

   to find the power series representation (centered at 0) of the function $g(x) = \frac{2}{1-4x}$. Give the interval of convergence of the new series.

5. $\boxed{\text{10 pts.}}$ Find the function represented by the series $\sum_{k=0}^{\infty} (\sqrt{x} - 7)^k$, and give the interval of convergence of the series.

6. Let $f(x) = \cos(5x)$.

   (a) $\boxed{\text{10 pts.}}$ Find the first four nonzero terms of the Maclaurin series for f.

   (b) $\boxed{\text{5 pts.}}$ Write the power series using summation notation.

   (c) $\boxed{\text{10 pts.}}$ Determine the interval of convergence for the series.

7. $\boxed{\text{10 pts.}}$ Evaluate $\lim_{x\to 0} \frac{3\tan^{-1}x - 3x + x^3}{x^5}$ using Taylor series.[1]

8. $\boxed{\text{10 pts.}}$ Use a Taylor series to approximate $\int_0^{0.15} \frac{\sin x}{x}dx$, retaining as many terms as needed to ensure the error is less than 10^{-4}.

9. $\boxed{\text{10 pts.}}$ Consider the parametric equations

$$x = (t+1)^2, \quad y = t+2; \quad -10 \leq t \leq 10.$$

   Eliminate the parameter to obtain an equation in x and y.

10. $\boxed{\text{10 pts.}}$ Give two alternative representations of the point $\left(8, \frac{2\pi}{3}\right)$ in polar coordinates.

11. $\boxed{\text{15 pts.}}$ Convert the equation $r\cos\theta = \sin(2\theta)$ to Cartesian coordinates, and describe the resulting curve.

[1] Do *not* use L'Hôpital's Rule.