MATH 141
SPRING 2022
EXAM 4

NAME:

1. [15 pts.] Approximate $\tan(-0.1)$ using an appropriate 3rd-order Taylor polynomial. Also compute the absolute error in the approximation assuming the exact value is given by a calculator.

2. [10 pts.] Use the remainder to find a bound on the error for the approximation

$$e^x \approx 1 + x + x^2/2$$

on the interval $[-\frac{1}{2}, \frac{1}{2}]$.

3. [10 pts. each] Determine the interval of convergence of the power series, making sure to test endpoints.

(a) $\sum \dfrac{x^n}{\sqrt{n^2+3}}$ (b) $\sum \left(1+\dfrac{1}{n}\right)^n (x+2)^n$ (c) $\sum (\ln n) x^n$

4. [10 pts.] Find the function represented by the series

$$\sum_{n=0}^{\infty} e^{-nx}.$$

5. [10 pts.] Let $f(x) = (1+x^2)^{-1/2}$. Find explicitly the first four nonzero terms of the Taylor series for f centered at 0.

6. [10 pts.] Use a Taylor series to estimate integral

$$\int_0^{0.1} \frac{\ln(1+x)}{x}\, dx$$

with an absolute error less than 10^{-5}.

7. [10 pts.] For the parametric equations

$$x = \sec^2 t - 1, \quad y = \tan t; \quad -\frac{\pi}{2} < t < \frac{\pi}{2},$$

eliminate the parameter to obtain a Cartesian equation of the form $y = f(x)$ or $x = g(y)$. State the domain of the function.

8. [10 pts.] Find parametric equations for a circle centered at $(2, 3)$ with radius 1, generated counter-clockwise.

9. [10 pts.] Convert the polar equation $r = e^{r\cos\theta} \csc\theta$ to Cartesian coordinates.

Alternating Series Estimation Theorem: If $\sum (-1)^{k+1} b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}.$$