Math 141
Spring 2022 Name:
Exam 2

1. [10 pts.] Evaluate the integral using a clever u-substitution:

$$\int_0^{\sqrt{3}} \frac{6x^3}{\sqrt{x^2+1}}\,dx.$$

2. [10 pts.] Use integration by parts to evaluate

$$\int_0^{\ln 2} xe^{-4x}\,dx.$$

3. [10 pts.] Use integration by parts to determine

$$\int \theta \sec^2\theta\,d\theta.$$

4. [10 pts.] Determine the trigonometric integral

$$\int \sin^5 2x\,dx.$$

5. [10 pts.] Use a trigonometric substitution to evaluate

$$\int_0^3 \frac{x^2}{x^2+9}\,dx.$$

6. [10 pts.] The length of the curve $y = f(x)$ for $a \le x \le b$ is defined to be

$$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx.$$

Find the length of the curve $y = ax^2$ from $x = 0$ to $x = 10$, where $a > 0$.

7. [10 pts. each] Use partial fractions to find the indefinite integral.

(a) $\displaystyle\int \frac{12}{(2s-1)(s-6)}\,ds$

(b) $\displaystyle\int \frac{x-5}{x^2(x+1)}\,dx$

8. [10 pts. each] Evaluate the improper integral, or show that it diverges.

(a) $\displaystyle\int_{-\infty}^{0} e^x \, dx$

(b) $\displaystyle\int_{0}^{32} \frac{1}{(y-1)^{6/5}} \, dy$

9. [10 pts.] Use the Comparison Theorem to determine whether the integral converges or diverges:

$$\int_1^\infty \frac{dx}{x^5 + x^3 + 1}.$$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$
- $\displaystyle\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
- $\displaystyle\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$
- $\displaystyle\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$
- $\displaystyle\int \tan x \, dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x \, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x \, dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x \, dx = -\ln|\csc x + \cot x| + c$