Math 141
Spring 2022
Exam 1

Name:

1. [10 pts.] Given $f(x) = 3x - x^{-3}$ and $x > 0$, find $(f^{-1})'(2)$ using the Inverse Function Theorem.

2. [10 pts. each] Find the derivative of each function.

   (a) $f(x) = \cos(\ln x)$

   (b) $g(t) = e^{\sqrt{t}+2}$

   (c) $h(x) = x^{(x^2)}$

   (d) $p(r) = \sec^{-1}(\sqrt{r^2+2})$

   (e) $\varphi(x) = \sinh(\tan^{-1} x)$

3. [10 pts.] Find an equation for the tangent line to $y = xe^{2x}$ at $x = \frac{1}{2}$.

4. [10 pts. each] Evaluate each integral.

   (a) $\displaystyle\int_4^5 \frac{s}{s^2-1}\,ds$

   (b) $\displaystyle\int_0^1 te^{-t^2}\,dt$

   (c) $\displaystyle\int \frac{\log_6 x}{x}\,dx$

   (d) $\displaystyle\int \frac{e^{2\sqrt{x}}}{\sqrt{x}}\,dx$

5. [10 pts.] Find the critical points of $f(x) = xe^{-x^2}$, and use the First Derivative Test to locate the local maximum and minimum values.

6. [10 pts. each] Evaluate the limit using L'Hôpital's Rule.

   (a) $\displaystyle\lim_{x\to 0^+} (3x)^{x/2}$

   (b) $\displaystyle\lim_{x\to\infty} \left(\frac{4x}{4x+5}\right)^{3x}$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$