Math 141
Spring 2020
Exam 1

Name:

1. [10 pts.] Find the derivative of the inverse of the function $f(x) = x^2 - 2x - 3$, $x \leq 1$, at the point $(12, -3)$ on the graph of f^{-1}. Do not find f^{-1}.

2. [10 pts. each] Find the derivative.

   (a) $\frac{d}{dx}(\ln|\sin x|)$

   (b) $\frac{d}{dx}(\cot e^x)$

   (c) $\frac{d}{dx}(3^{x^2-x})$

   (d) $\frac{d}{dx}(1+x^2)^{\sin x}$

   (e) $\frac{d}{dx}(\log_8|\tan x|)$

   (f) $\frac{d}{dx}(\sin^{-1}(\ln x))$

   (g) $\frac{d}{dt}(\csc^{-1}(\tan e^t))$

   (h) $\frac{d}{dz}(\sqrt{\operatorname{sech} 8z})$

3. [15 pts.] Find an equation of the tangent line to $y = 2^{\sin x}$ at $x = \pi$.

4. [10 pts. each] Evaluate each integral.

   (a) $\int_{-2}^{3} \frac{12}{18-5t}\,dt$

   (b) $\int_{0}^{\pi/2} \frac{\sin x}{1+\cos x}\,dx$

   (c) $\int \frac{\log_2 z^{10}}{z}\,dz$

   (d) $\int_{1}^{2} (1+\ln x)x^x\,dx$

   (e) $\int \frac{e^x}{e^{2x}+4}\,dx$

   (f) $\int_{0}^{1} \cosh^3 3y \sinh 3y\,dy$

5. 10 pts. Find the length of the curve $x = 2e^{\sqrt{2}y} + \frac{1}{16}e^{-\sqrt{2}y}$, for $0 \le y \le \frac{\ln 2}{\sqrt{2}}$.

6. 10 pts. each Evaluate the limit, using L'Hôpital's Rule when applicable.

(a) $\displaystyle\lim_{x\to\infty}\left(3+\frac{2}{x}\right)^{\ln x}$

(b) $\displaystyle\lim_{x\to 0}(x+\cos x)^{1/x}$

(c) $\displaystyle\lim_{x\to 0}(1+a^x)^{b/x}$, for $a, b > 0$

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|+c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$