MATH 141
SPRING 2019
EXAM 4

NAME:

1. $\boxed{\text{10 pts. each}}$ Determine the interval of convergence of the power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$

2. $\boxed{\text{15 pts.}}$ Find a power series representation for the function

$$f(x) = \frac{x^2}{x^4+16},$$

and determine the interval of convergence.

3. $\boxed{\text{10 pts.}}$ Use a Taylor series to estimate integral

$$\int_0^{0.2} x \ln(1+x^2)\, dx$$

with an absolute error less than 10^{-5}.

4. $\boxed{\text{10 pts.}}$ Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = \sqrt[3]{x}$ centered at 8.

5. $\boxed{\text{10 pts.}}$ Use the binomial series to expand the function $\sqrt[4]{1-x}$ as a power series, and state the radius of convergence.

6. $\boxed{\text{10 pts.}}$ Find the exact length of the curve $y = \ln(\sec x)$, $0 \le x \le \pi/4$.

7. $\boxed{\text{15 pts.}}$ Eliminate the parameter to find a Cartesian equation of the curve given by

$$x = \tan^2 t, \quad y = \sec t, \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

Sketch the curve and indicate with an arrow the direction in which the curve is traced as t increases.

8. $\boxed{\text{15 pts.}}$ Find dy/dx and d^2y/dx^2 for the curve

$$x = t - \ln t, \quad y = t + \ln t.$$

For what values of t is the curve concave upward?

9. $\boxed{\text{10 pts.}}$ Find the slope of the tangent line to the polar curve $r = 2 + \sin\theta$ at the point corresponding to $\theta = \pi/4$.

10. $\boxed{\text{10 pts.}}$ Find the area of the region enclosed by one loop of the curve $r = 2\sin 5\theta$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \le b_{k+1} \le b_k$ for all k, then $R_n \le b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \le 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \le 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}.$$