MATH 141
SPRING 2019 NAME:
EXAM 3

1. [10 pts. each] Find the limit of each sequence, or show that the limit does not exist.

   (a) $\left(n \tan \frac{\pi}{n}\right)_{n=1}^{\infty}$

   (b) $\left(\sqrt{n^4 - 2n} - n^2\right)_{n=2}^{\infty}$

2. [10 pts.] Determine whether the sequence

$$a_n = \frac{1-n}{2+n}$$

   is increasing, decreasing, or not monotonic. Is the sequence bounded?

3. [10 pts.] Evaluate the geometric series $\sum_{n=1}^{\infty} \frac{4}{6^n}$.

4. [10 pts.] Either show the telescoping series

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$$

   diverges, or find a formula for the nth partial sum s_n and evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

5. [10 pts.] Find all values of c for which the series converges:

$$\sum_{n=1}^{\infty} \left(\frac{c}{n} - \frac{1}{n+1}\right).$$

   (Hint: the Integral Test may help.)

6. [10 pts. each] Determine whether the series converges or diverges, using an appropriate test and justifying all work. Arguments must be clear and thorough.

   (a) $\sum_{n=0}^{\infty} \frac{4}{2+3^n n}$

   (b) $\sum_{n=1}^{\infty} \frac{4^n}{n^2}$

   (c) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} n^{-1/n}$

(f) $1 + \dfrac{1 \cdot 3}{3!} + \dfrac{1 \cdot 3 \cdot 5}{5!} + \dfrac{1 \cdot 3 \cdot 5 \cdot 7}{7!} + \cdots$

7. [10 pts. each] Use the Alternating Series Test to show the series converges, or use some other test to show it diverges. If the series converges, use any test to determine whether it converges absolutely or conditionally.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{5/4}}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n n}{\ln n}$

Some Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$