1. [10 pts.] Find the integral

$$\int \frac{1}{x^{-1}+1}\, dx$$

2. [10 pts. each] Use integration by parts to determine each of the following.

(a) $\int e^x \sin x \, dx$

(b) Let $\mathcal{R}$ be the region bounded by $f(x) = x \ln x$ and the x-axis on $[1, e^2]$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

3. [10 pts. each] Find each indefinite trigonometric integral.

(a) $\int \sin^5 x \cos^{-2} x \, dx$

(b) $\int \cot^4 x \, dx$

(c) $\int \tan^9 x \sec^4 x \, dx$

4. [10 pts. each] Use a trigonometric substitution to find the indefinite integral

(a) $\int_{1/2}^{1} \frac{\sqrt{1-y^2}}{y^2}\, dy$

(b) $\int \frac{1}{x^2\sqrt{9x^2-1}}\, dx, \quad x > \frac{1}{3}$

5. [10 pts. each] Use partial fractions to find the indefinite integral

(a) $\int \frac{8}{(y-4)^2(y+3)}\, dy$

(b) $\int \frac{2}{(x-4)(x^2+2x+6)}\, dx$

6. [10 pts.] Evaluate the improper integral or state that it diverges:

$$\int_2^\infty \frac{\cos(\pi/x)}{x^2}\, dx$$

7. [10 pts.] Evaluate the improper integral, or state that it diverges:

$$\int_0^1 x \ln x \, dx.$$

8. [10 pts.] Let $\mathcal{R}$ be the region on the xy-plane bounded by

$$f(x) = \sqrt{\frac{x+1}{x^3}}$$

and the x-axis on the interval $[1, \infty)$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

FORMULAS & DEFINITIONS

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1 - x^2}}$

- $(\tan^{-1} x)' = \dfrac{1}{1 + x^2}$

- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2 - 1}}$

- $\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

- $\displaystyle\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

- $\displaystyle\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$

- $\displaystyle\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$

- $\displaystyle\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$

- $\displaystyle\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n - 1} - \int \tan^{n-2} x \, dx$

- $\displaystyle\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n - 1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$

- $\displaystyle\int \tan x \, dx = \ln|\sec x| + c$

- $\displaystyle\int \cot x \, dx = \ln|\sin x| + c$

- $\displaystyle\int \sec x \, dx = \ln|\sec x + \tan x| + c$

- $\displaystyle\int \csc x \, dx = -\ln|\csc x + \cot x| + c$