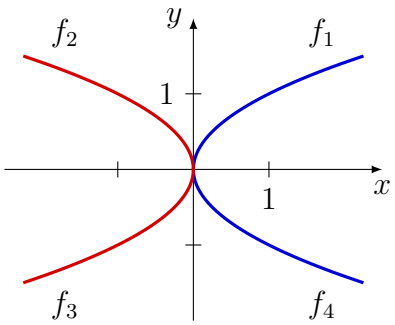
MATH 141
SPRING 2018 NAME:
EXAM 1

1. [10 pts. each] The equation $y^4 = 4x^2$ gives rise to four one-to-one functions f_1, f_2, f_3, f_4, where the graph of f_k is in the kth quadrant for each $1 \le k \le 4$ (see figure below).
   (a) Find the domain and a formula for each function f_k.
   (b) Find the inverse of each function and write it as $y = f_k^{-1}(x)$.



2. [15 pts.] Consider the function $f(x) = x - \pi + \cos x$, $0 < x < 2\pi$. Show that f is one-to-one and hence has an inverse, then find $(f^{-1})'(-1)$ using the Inverse Function Theorem.

3. [10 pts. each] Find the derivative of each function.
   (a) $y = \dfrac{\ln x}{\ln x + 1}$
   (b) $f(x) = \sin(\cos(e^x))$
   (c) $g(x) = (\cot x)^{x-1}$
   (d) $h(t) = t^{(t^{1/2})}$
   (e) $\ell(x) = 4\log_3(1 - x^3)$
   (f) $\varphi(u) = \tan^{-1}(4u - 4)$
   (g) $y = x^2 \cosh^3 5x$

4. [10 pts.] Find an equation of the line tangent to $y = 2e^x + 8$ at the point where $x = \ln 6$.

5. [10 pts. each] Determine each indefinite integral.
   (a) $\displaystyle\int x^5 e^{x^6}\,dx$
   (b) $\displaystyle\int \frac{1}{(x \ln x)\ln(\ln x)}\,dx$
   (c) $\displaystyle\int \frac{e^{\cos x}}{\csc x}\,dx$

6. [10 pts. each] Evaluate each definite integral.

(a) $\displaystyle\int_{-2}^{2} \frac{e^{t/2}}{e^{t/2}+1}\,dt$

(b) $\displaystyle\int_{0}^{3/2} \frac{1}{\sqrt{36-4x^2}}\,dx$

(c) $\displaystyle\int_{0}^{1} \cosh^3 4x \sinh 4x\,dx$

7. [10 pts.] Evaluate the limit using L'Hôpital's Rule: $\displaystyle\lim_{x\to\infty} (x^2+2)^{1/\ln x}$.

Formulas & Definitions

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$