

1. 10 pts. Approximate $\cos(2^\circ)$ using a 4th-order Taylor polynomial centered at 0. (Note: it will be necessary to convert the angle to radians.)

2. 10 pts. each Determine the interval of convergence of the power series, making sure to test endpoints.

(a) $\sum \frac{x^n}{\sqrt{n^2 + 3}}$ (b) $\sum \left(2 + \frac{2}{n}\right)^n (x - 1)^n$ (c) $\sum (\ln n)x^n$

3. 10 pts. Find the function represented by the series

$$\sum_{n=0}^{\infty} \left(\frac{x^2 - 1}{2}\right)^n.$$

4. 5 pts. each Let $f(x) = (1 + x^3)^{-1/3}$.

(a) Find explicitly the first four nonzero terms of the Taylor series for f centered at 0.

(b) Write the Taylor series using summation notation.

(c) Find the radius of convergence and interval of convergence.

5. 10 pts. Use a Taylor series to estimate integral

$$\int_0^{0.1} \frac{\ln(1+x)}{x} dx$$

with an absolute error less than 10^{-5} .

6. 10 pts. For the parametric equations

$$x = \sec^2 t - 1, \quad y = \tan t; \quad -\frac{\pi}{2} < t < \frac{\pi}{2},$$

eliminate the parameter to obtain a Cartesian equation of the form $y = f(x)$ or $x = g(y)$. State the domain of the function.

7. 10 pts. An object moves along a straight path from the point $(3, -4)$ at time $t = 0$ to the point $(2, 0)$ at time $t = 3$. Find a parametric description of the object's path.

8. 10 pts. Convert the polar equation $r = e^{r \cos \theta} \csc \theta$ to Cartesian coordinates.

9. 10 pts. Find the area inside one loop of the lemniscate $r^2 = 4 \sin 2\theta$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k , then $R_n \leq b_{n+1}$ for all n .

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}.$$