MATH 141
SPRING 2017 NAME:
EXAM 2

1. [10 pts.] Find the area of the region bounded by the curves:

$$y = \frac{x^3}{x^2+1} \quad \text{and} \quad y = \frac{8x}{x^2+1}.$$

2. [10 pts.] Use integration by parts to evaluate

$$\int_0^{1/\sqrt{2}} y \tan^{-1} y^2 \, dy.$$

3. [10 pts.] Use integration by parts to determine

$$\int x \sin x \cos x \, dx.$$

4. [10 pts.] Determine the trigonometric integral

$$\int \sin^3 \beta \cos^5 \beta \, d\beta.$$

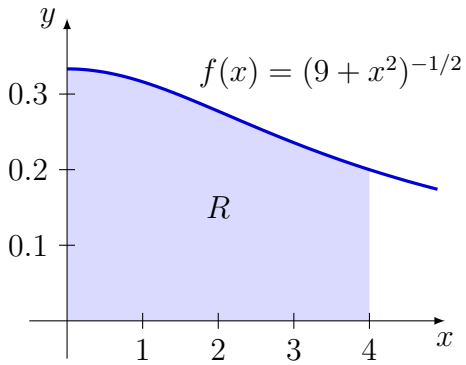
5. [10 pts.] Use a trigonometric substitution to evaluate

$$\int_6^{6\sqrt{3}} \frac{q^2}{(q^2+36)^2} dq.$$

6. [10 pts. each]

(a) Find the area of the region R bounded by the curves $f(x) = (9+x^2)^{-1/2}$, $x = 0$, $x = 4$, and $y = 0$. See figure below.

(b) Find the volume of the solid generated when R is revolved about the x-axis.



7. [10 pts. each] Use partial fractions to find the indefinite integral.

(a) $\displaystyle\int \frac{12}{(r-4)(r+3)}\,dr$

(b) $\displaystyle\int \frac{x-5}{x^2(x+1)}\,dx$

8. [10 pts. each] Evaluate the improper integral, or show that it diverges.

(a) $\displaystyle\int_{-\infty}^{-2} \frac{2}{t^2-1}\,dt$

(b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2/2}dx$

(c) $\displaystyle\int_{1}^{8} \frac{1}{(z-3)^{4/3}}\,dz$

Formulas

- $\tan x = y \;\Leftrightarrow\; \tan^{-1} y = x, \quad \text{for } x \in (-\pi/2, \pi/2)$
- $\cot x = y \;\Leftrightarrow\; \cot^{-1} y = x, \quad \text{for } x \in (0, \pi)$
- $\sec x = y \;\Leftrightarrow\; \sec^{-1} y = x, \quad \text{for } x \in [0, \pi/2) \cup (\pi/2, \pi]$
- $\csc x = y \;\Leftrightarrow\; \csc^{-1} y = x, \quad \text{for } x \in [-\pi/2, 0) \cup (0, \pi/2]$
- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + c$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\!\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\!\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c, \qquad \int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c, \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + c$