MATH 141
SPRING 2017 NAME:
EXAM 1

1. [10 pts.] Suppose the slope of the curve $y = f^{-1}(x)$ at $(4, 7)$ is $\frac{4}{5}$. Find $f'(7)$.

2. [10 pts. each] Find the derivative of each function.

   (a) $f(x) = \ln\left(\dfrac{x+1}{x-1}\right)$

   (b) $g(x) = (\ln x)^{x^2}$

   (c) $h(t) = (\tan t)^{\tan t}$

   (d) $r(x) = \log_7 \sqrt[3]{8x}$

   (e) $\varphi(z) = \sec^{-1}(\ln z)$

   (f) $y = \cosh^4(e^{4x})$

3. [10 pts.] Find an equation of the line tangent to $y = x^{\sin x}$ at the point $x = 1$.

4. [10 pts. each] Determine each indefinite integral.

   (a) $\displaystyle\int \frac{e^x}{4e^x + 6}\,dx$

   (b) $\displaystyle\int \frac{1}{(x \ln x)\ln(\ln x)}\,dx$

   (c) $\displaystyle\int \frac{e^{\sin x}}{\sec x}\,dx$

   (d) $\displaystyle\int \frac{\sinh t}{1 + \cosh t}dt.$

5. [10 pts. each] Evaluate each definite integral.

   (a) $\displaystyle\int_1^{2e} \frac{3^{\ln x}}{x}\,dx$ (b) $\displaystyle\int_{-\ln\sqrt{3}}^{0} \frac{e^x}{1 + e^{2x}}\,dx$

6. [10 pts.] Evaluate the limit, using L'Hôpital's Rule if necessary: $\displaystyle\lim_{x \to 2^+} x^{3/(2-x)}$

Formulas & Definitions

Formulas

- $\tan x = y \;\Leftrightarrow\; \tan^{-1} y = x, \quad \text{for } x \in (-\pi/2, \pi/2)$
- $\cot x = y \;\Leftrightarrow\; \cot^{-1} y = x, \quad \text{for } x \in (0, \pi)$
- $\sec x = y \;\Leftrightarrow\; \sec^{-1} y = x, \quad \text{for } x \in [0, \pi/2) \cup (\pi/2, \pi]$
- $\csc x = y \;\Leftrightarrow\; \csc^{-1} y = x, \quad \text{for } x \in [-\pi/2, 0) \cup (0, \pi/2]$
- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int a^x \, dx = \frac{a^x}{\ln a} + c$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\!\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a}\tan^{-1}\!\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}} \, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x \, dx$
- $\displaystyle\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x \, dx$
- $\displaystyle\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$
- $\displaystyle\int \sec^n x \, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x \, dx$
- $\displaystyle\int \tan x \, dx = \ln|\sec x| + c, \qquad \int \cot x \, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x \, dx = \ln|\sec x + \tan x| + c, \qquad \int \csc x \, dx = -\ln|\csc x + \cot x| + c$