Math 141
Spring 2016
Exam 4

Name:

1. $\boxed{\text{10 pts.}}$ Approximate the quantity $\sqrt[3]{126}$ using a 3rd-order Taylor polynomial centered at 125.

2. $\boxed{\text{10 pts. each}}$ Determine the interval of convergence of the power series, making sure to test endpoints.

   (a) $\sum \frac{n^3 x^{4n}}{n!}$ (b) $\sum \frac{(-1)^{n-1} x^n}{n^3}$ (c) $\sum \frac{(-2)^n}{\sqrt[4]{n}}(x-1)^n$

3. $\boxed{\text{10 pts.}}$ Find the function represented by the series

$$\sum_{n=0}^{\infty} \left(\frac{3}{2x^2+1} \right)^n,$$

   and give the interval of convergence.

4. Let $f(x) = 1/x$.

   (a) $\boxed{\text{10 pts.}}$ Find the first four nonzero terms of the Taylor series for f centered at 2.

   (b) $\boxed{\text{5 pts.}}$ Write the Taylor series using summation notation.

5. $\boxed{\text{10 pts.}}$ Use a Taylor series to approximate the value of the definite integral

$$\int_0^{1/2} e^{-x^2}\, dx$$

   with an absolute error less than 10^{-8}.

6. $\boxed{\text{10 pts.}}$ Consider the parametric equations

$$x = \sqrt[5]{t} - 2, \quad y = t + 1; \quad 0 \le t \le 32.$$

   Eliminate the parameter to obtain an equation of the form $y = f(x)$. What is the domain of f?

7. $\boxed{\text{10 pts.}}$ Find a parametric description of the line segment from the point $(8, 2)$ to the point $(-2, -3)$.

8. $\boxed{\text{10 pts.}}$ Convert the polar equation $r = 2\sin\theta + 2\cos\theta$ to Cartesian coordinates.

9. $\boxed{\text{10 pts.}}$ Find the area of the region inside the limaçon $r = 2 + \cos\theta$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1 \text{ (Geometric Series)}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

Some Trigonometric Identities:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$