Math 141
Spring 2016
Exam 3

Name:

1. 5 pts. each Consider the sequence $(1, -4, 9, -16, 25, \ldots)$.
   (a) Find a recurrence relation that generates the sequence.
   (b) Find an explicit formula for the nth term of the sequence.

2. 10 pts. Find the limit of the sequence

$$a_n = \frac{3^{n+1} + 3}{3^n},$$

or determine that it does not exist.

3. 10 pts. Find the limit of the sequence $a_n = (1/n)^{1/n}$, or determine that it does not exist.

4. 15 pts. Write the repeating decimal $1.\overline{25} = 1.2525252525...$ first as a geometric series, and then evaluate the series as a fraction (i.e. a ratio of integers).

5. 15 pts. Let p be a positive integer. For the telescoping series

$$\sum_{k=1}^{\infty} \frac{1}{(k+p)(k+p+1)},$$

find a formula for the nth term of the sequence of partial sums (s_n), and then evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

6. 5 pts. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^e}{n^\pi}$$

converges or diverges, stating the reason.

7. 10 pts. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

converges or diverges using either the Divergence Test or Integral Test.

8. 10 pts. Use either the Direct Comparison Test or Limit Comparison Test to determine whether

$$\sum_{n=1}^{\infty} \frac{1}{2n - \sqrt{n}}$$

converges or diverges.

9. 10 pts. Use the Ratio Test to determine whether

$$\sum_{n=1}^{\infty} \frac{n^{100}}{(n+1)!}$$

converges or diverges.

10. 10 pts. Use the Root Test to determine whether

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{2n^2}$$

converges or diverges.

11. 10 pts. Use the Alternating Series Test to show the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{n^2-1}{4n^2+9}$$

converges, or use another test to show it diverges.