

1. 10 pts. Find the integral

$$\int \frac{1}{x^{-1} + 1} dx$$

2. 10 pts. each Use integration by parts to determine each of the following.

(a) $\int e^x \sin x dx$

- (b) Let \mathcal{R} be the region bounded by $f(x) = x \ln x$ and the x -axis on $[1, e^2]$. Find the volume of the solid generated by revolving \mathcal{R} about the x -axis.

3. 10 pts. each Find each indefinite trigonometric integral.

(a) $\int \sin^5 x \cos^{-2} x dx$

(b) $\int \cot^4 x dx$

(c) $\int \tan^9 x \sec^4 x dx$

4. 10 pts. each Use a trigonometric substitution to find the indefinite integral

(a) $\int_{1/2}^1 \frac{\sqrt{1-y^2}}{y^2} dy$

(b) $\int \frac{1}{x^2 \sqrt{9x^2 - 1}} dx, \quad x > \frac{1}{3}$

5. 10 pts. each Use partial fractions to find the indefinite integral

(a) $\int \frac{8}{(y-4)^2(y+3)} dy$

(b) $\int \frac{2}{(x-4)(x^2+2x+6)} dx$

6. 10 pts. Evaluate the improper integral:

$$\int_2^{\infty} \frac{1}{(t+2)^2} dt$$

7. 10 pts. Evaluate the improper integral, or show that it diverges:

$$\int_{-1}^1 \ln(y^2) dy.$$

FORMULAS & DEFINITIONS

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan \theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot \theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec \theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc \theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
6. $(\tan^{-1} x)' = \frac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
7. $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, -1) \cup (1, \infty)$
8. $\int a^x dx = \frac{a^x}{\ln a} + c$, for $a \in (0, 1) \cup (1, \infty)$
9. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
10. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
11. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
12. $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
13. $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
14. $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$, $n \neq 1$
15. $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$, $n \neq 1$
16. $\int \tan x dx = -\ln |\cos x| + c = \ln |\sec x| + c$
17. $\int \cot x dx = \ln |\sin x| + c$
18. $\int \sec x dx = \ln |\sec x + \tan x| + c$
19. $\int \csc x dx = -\ln |\csc x + \cot x| + c$