

1. 10 pts. For the function $f(x) = -x^2 + 8$, find the slope of the line tangent to the graph of f^{-1} at the point $(4, 2)$.

2. 10 pts. each Find the derivative of each function.

(a) $f(x) = \cot(e^x)$

(b) $g(x) = \ln(\ln x)$

(c) $h(x) = (2x + 1)^{2x}$

(d) $x(t) = \sec(\sin^{-1} 5t)$

(e) $v(y) = \tan^{-1}\left(\frac{1}{y^2 + 1}\right)$

(f) $\varphi(x) = \tanh^3(x^2)$

3. 10 pts. each Determine each indefinite integral.

(a) $\int (2e^{-6x} + e^{9x}) dx$

(b) $\int \frac{9}{4 - 9y} dy$

(c) $\int x^7 8^{x^8} dx$

(d) $\int \frac{\sinh t}{1 + \cosh t} dt.$

4. 10 pts. each Evaluate each definite integral.

(a) $\int_1^2 (1 + \ln x)x^x dx$

(b) $\int_0^{\ln \sqrt{3}} \frac{e^x}{1 + e^{2x}} dx$

5. 10 pts. Evaluate the limit using L'Hôpital's Rule:

$$\lim_{x \rightarrow 1^+} (\sqrt{x-1})^{2 \sin \pi x}$$

FORMULAS & DEFINITIONS

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan \theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot \theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec \theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc \theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
6. $(\tan^{-1} x)' = \frac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
7. $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, -1) \cup (1, \infty)$
8. $\int b^x dx = \frac{1}{\ln b} b^x + c$, for $b \in (0, 1) \cup (1, \infty)$
9. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c$, for $a \in (0, \infty)$
10. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$, for $a \neq 0$
11. $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + c$, for $a \in (0, \infty)$
12. $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
13. $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
14. $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$, $n \neq 1$
15. $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$, $n \neq 1$
16. $\int \tan x dx = -\ln |\cos x| + c = \ln |\sec x| + c$
17. $\int \cot x dx = \ln |\sin x| + c$
18. $\int \sec x dx = \ln |\sec x + \tan x| + c$
19. $\int \csc x dx = -\ln |\csc x + \cot x| + c$