## Math 141 Spring 2015 Exam 3

## NAME:

- 1. 10 pts. each The first six terms of the sequence  $(a_n)_{n=1}^{\infty}$  are  $-4, 7, -10, 13, -16, 19, \ldots$ 
  - (a) Assuming the pattern established by the first six terms continues indefinitely, find a recurrence relation that generates  $(a_n)_{n=1}^{\infty}$ .
  - (b) Find an explicit formula for the nth term of the sequence.
- 2. 10 pts. A radioactive isotope transmutes 18% of its mass to another element every 12 years. Let  $M_n$  be the mass of the isotope at the end of the *n*th twelve-year period, where the initial mass of the material is  $M_0 = 80$  grams. Find an explicit formula for  $M_n$ , the *n*th term of the sequence  $(M_n)_{n=0}^{\infty}$ .
- 3. 10 pts. Find the limit of the sequence

$$\left(\frac{4n-5n^6}{11n^6-9}\right)_{n=1}^{\infty}$$

or determine that the limit does not exist.

4. 10 pts. Evaluate the geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{6}\right)^n 3^{4-n},$$

if it converges.

5. 10 pts. For the telescoping series

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+4)},$$

find a formula for the *n*th term of the sequence of partial sums  $(s_n)_{n=1}^{\infty}$ , then evaluate  $\lim_{n\to\infty} s_n$  to obtain the value of the series.

- 6. 10 pts. each Determine whether the series converges or diverges using the indicated test, or state that the test is inconclusive.
  - (a)  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ , using the Integral Test. (b)  $\sum_{n=0}^{\infty} \frac{1}{n+1000}$ , using the Divergence Test. (c)  $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$ , using the Ratio Test.

(d) 
$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{2n^2}$$
, Root Test.  
(e)  $\sum_{n=1}^{\infty} \frac{1}{n^2+8}$ , using the Comparison Test or Limit Comparison Test.  
(f)  $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+1}}{\sqrt{n^3+2}}$ , using the Comparison Test or Limit Comparison Test.

7. 10 pts. each Use the Alternating Series Test to show the series converges, or use another test to show it diverges.

(a) 
$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$$
  
(b)  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+6}$