MATH 141
SPRING 2015 NAME:
EXAM 2

1. 10 pts. each Use division to evaluate the integral

$$\int_2^4 \frac{x^2+2}{x-1}\,dx$$

2. 10 pts. each Use integration by parts to determine each of the following.

(a) $\int \frac{x}{\sqrt{x+1}}\,dx$

(b) Let $\mathcal{R}$ be the region bounded by $f(x) = x\ln x$ and the x-axis on $[1, e^2]$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

3. 10 pts. each Find each indefinite trigonometric integral.

(a) $\int (\cos^3 x)\sqrt{\sin x}\,dx$

(b) $\int \frac{\sec^2 z}{\tan^5 z}\,dz$

(c) $\int e^x \sec(e^x+1)\,dx$

4. 10 pts. each Use a trigonometric substitution to find the indefinite integral

(a) $\int_0^{1/3} \frac{1}{(9x^2+1)^{3/2}}\,dx$

(b) $\int \sqrt{169-t^2}\,dt$

5. 10 pts. each Use partial fractions to find the indefinite integral

(a) $\int \frac{2}{x^3+x^2}\,dx$

(b) $\int \frac{2}{(x-4)(x^2+2x+6)}\,dx$

6. 10 pts. Let $\mathcal{R}$ be the region on the xy-plane bounded by

$$f(x) = \sqrt{\frac{x+1}{x^3}}$$

and the x-axis on the interval $[1, \infty)$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

7. 10 pts. Use integration by parts to evaluate the improper integral

$$\int_0^1 x\ln x\,dx.$$

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
6. $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
7. $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
8. $\displaystyle\int b^x\,dx = \frac{1}{\ln b} b^x + c$, for $b \in (0, 1) \cup (1, \infty)$
9. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
10. $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
11. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
12. $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
13. $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
14. $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx, \;\; n \neq 1$
15. $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx, \;\; n \neq 1$
16. $\displaystyle\int \tan x\,dx = -\ln|\cos x| + c = \ln|\sec x| + c$
17. $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
18. $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
19. $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$