MATH 141
SPRING 2014 NAME:
EXAM 3

1. [10 pts. each] Find the limit of the sequence, or explain why the limit does not exist.

   (a) $\left\{\dfrac{12n^5 - 4n^2}{3 - 5n - 9n^5}\right\}$

   (b) $a_n = (-1)^n \sqrt[n]{n}$

2. [10 pts.] Evaluate the geometric series, if it converges: $\displaystyle\sum_{k=1}^{\infty} 2^{-3k}$.

3. [10 pts.] For the telescoping series

$$\sum_{k=1}^{\infty}\left(\frac{1}{k+5} - \frac{1}{k+6}\right),$$

   find a formula for the nth term of the sequence of partial sums $\{s_n\}$, then evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

4. [10 pts. each] Determine whether the series converges or diverges using one of the indicated tests.

   (a) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\sqrt{k^2+25}}$, Divergence or Integral Test

   (b) $\displaystyle\sum_{k=1}^{\infty} ke^{-2k^2}$, Divergence or Integral Test

   (c) $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k^{99}}$, Ratio Test

   (d) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{2k^2}$, Root Test

   (e) $\displaystyle\sum_{k=1}^{\infty} \frac{\sin^2 k}{k\sqrt{k}}$, either comparison test

   (f) $\displaystyle\sum_{k=1}^{\infty} \frac{k^7}{k^9+3}$, either comparison test

5. [10 pts. each] Use the Alternating Series Test to show the series converges, or use another test to show it diverges.

   (a) $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k\ln^2 k}$

   (b) $\displaystyle\sum_{k=1}^{\infty} (-1)^k\left(1 - \frac{2}{k}\right)$