MATH 141
SPRING 2014 NAME:
EXAM 2

1. [10 pts. each] Use integration by parts (possibly more than once) to find each integral.

   (a) $\displaystyle\int_{1/2}^{\sqrt{3}/2} \sin^{-1} z \, dz$

   (b) $\displaystyle\int x^2 \ln^2 x \, dx$

2. [10 pts. each] Find each indefinite trigonometric integral.

   (a) $\displaystyle\int \sin^7(x) \cos^3(x) \, dx$

   (b) $\displaystyle\int \frac{\csc^4 t}{\cot^2 t} \, dt$

   (c) $\displaystyle\int_0^{\pi/2} \sqrt{1 - \cos 2x} \, dx$

3. [10 pts. each] Use a trigonometric substitution to find the indefinite integral

   (a) $\displaystyle\int_0^{1/3} \frac{1}{(9x^2+1)^{3/2}} \, dx$

   (b) $\displaystyle\int \sqrt{169 - t^2} \, dt$

4. [10 pts. each] Use partial fractions to find the indefinite integral

   (a) $\displaystyle\int \frac{2}{x^3 + x^2} \, dx$

   (b) $\displaystyle\int \frac{2}{(x-4)(x^2+2x+6)} \, dx$

5. [10 pts.] Evaluate the improper integral

$$\int_2^{\infty} \frac{\cos(\pi/x)}{x^2} dx$$

   or state that it diverges.

6. [10 pts.] Let $\mathcal{R}$ be the region on the xy-plane bounded by

$$f(x) = \sqrt{\frac{x+1}{x^3}}$$

   and the x-axis on the interval $[1, \infty)$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

7. [10 pts.] Use integration by parts to evaluate the improper integral

$$\int_0^1 x \ln x \, dx.$$

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
6. $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
7. $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
8. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
9. $\displaystyle\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
10. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
11. $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
12. $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
13. $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx, \;\; n \neq 1$
14. $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx, \;\; n \neq 1$
15. $\displaystyle\int \tan x\, dx = -\ln|\cos x| + c = \ln|\sec x| + c$
16. $\displaystyle\int \cot x\, dx = \ln|\sin x| + c$
17. $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c$
18. $\displaystyle\int \csc x\, dx = -\ln|\csc x + \cot x| + c$