MATH 141
SPRING 2014 NAME:
EXAM 1

1. [10 pts.] Given that $f(x) = 2x^3 + x - 12$, find $(f^{-1})'(6)$.

2. [10 pts.] Find all the inverses associated with $f(x) = (x-6)^2$, and state their domains.

3. [10 pts. each] Find the derivative of each function.
   (a) $f(x) = \ln(e^{2x} + 3)$
   (b) $g(x) = x^{\ln(x^5)}$
   (c) $h(x) = (\sin x)^{\tan x}$
   (d) $k(x) = 7\log_3(4 - x^5)$
   (e) $\ell(x) = \sec^{-1}(e^{-2x})$
   (f) $p(x) = \cot^{-1}(\sqrt{x})$

4. [10 pts. each] Determine each indefinite integral.
   (a) $\displaystyle\int (3e^{-8x} - 8e^{11x})\,dx$
   (b) $\displaystyle\int \frac{9}{4 - 9y}\,dy$
   (c) $\displaystyle\int x^3 10^{x^4}\,dx$

5. [10 pts. each] Evaluate each definite integral.
   (a) $\displaystyle\int_1^{3e} \frac{e^{\ln(x)}}{2x}\,dx$
   (b) $\displaystyle\int_2^{2\sqrt{3}} \frac{5}{z^2 + 4}\,dz$

6. [10 pts.] Evaluate the limit using L'Hôpital's Rule:

$$\lim_{x\to\infty} \left(\frac{2}{3x}\right)^{8/x}$$

7. [10 pts.] Use limit methods to determine which of the two functions grows faster: x^{50}, 1.0005^x.

Formulas & Definitions

1. $\theta = \tan^{-1} x \Leftrightarrow x = \tan\theta$, for $\theta \in (-\pi/2, \pi/2)$
2. $\theta = \cot^{-1} x \Leftrightarrow x = \cot\theta$, for $\theta \in (0, \pi)$
3. $\theta = \sec^{-1} x \Leftrightarrow x = \sec\theta$, for $\theta \in [0, \pi/2) \cup (\pi/2, \pi]$
4. $\theta = \csc^{-1} x \Leftrightarrow x = \csc\theta$, for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$
5. $(b^x)' = b^x \ln b$, for $b \in (0,1) \cup (1, \infty)$
6. $(\log_b |x|)' = \dfrac{1}{x \ln b}$, for $x \neq 0$
7. $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$, for $x \in (-1, 1)$
8. $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$, for $x \in (-\infty, \infty)$
9. $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$, for $x \in (-\infty, 1) \cup (1, \infty)$
10. $\displaystyle\int b^x\, dx = \frac{1}{\ln b} b^x + c$, for $b \in (0,1) \cup (1, \infty)$
11. $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$, for $a \in (0, \infty)$
12. $\displaystyle\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$, for $a \neq 0$
13. $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$, for $a \in (0, \infty)$
14. $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
15. $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
16. $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx, \;\; n \neq 1$
17. $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx, \;\; n \neq 1$
18. $\displaystyle\int \tan x\, dx = -\ln|\cos x| + c = \ln|\sec x| + c$
19. $\displaystyle\int \cot x\, dx = \ln|\sin x| + c$
20. $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c$
21. $\displaystyle\int \csc x\, dx = -\ln|\csc x + \cot x| + c$