Math 141
Exam #4
Spring 2011

Name:

1. 15 pts. Determine the radius of convergence of $\sum_{k=0}^{\infty} \frac{(x-1)^k k^k}{(k+1)^k}$, then test the endpoints to determine the interval of convergence.

2. 15 pts. Find the power series representation for $f(x) = \frac{1}{1+x^2}$, centered at 0, using known power series. Give the interval of convergence for the resulting series.

3. 10 pts. each Let $f(x) = e^{-3x}$.
   (a) Find the first four nonzero terms of the Maclaurin series for f.
   (b) Write the power series using summation notation.
   (c) Determine the interval of convergence for the series.

4. 15 pts. Use a Taylor series to approximate $\int_0^{0.2} \sin(x^2)dx$, retaining as many terms as needed to ensure the error is less than 10^{-4}.

5. 10 pts. Consider the parametric equations $x = (t+1)^2$, $y = t+2$; $-10 \le t \le 10$. Eliminate the parameter to obtain an equation in x and y.

6. 10 pts. Give two alternative representations of the point $\left(3, \frac{2\pi}{3}\right)$ in polar coordinates.

7. 15 pts. Convert the equation $r\cos\theta = \sin(2\theta)$ to Cartesian coordinates, and describe the resulting curve.

SOME POSSIBLY USEFUL THINGS:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$, for $|x| < 1$

$\frac{1}{1+x} = \sum_{k=0}^{\infty} (-1)^k x^k$, for $|x| < 1$

Remainder Theorem: Let $R_n = |S - S_n|$ be the remainder in approximating the value of a convergent alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ by the sum of its first n terms. Then $R_n \le a_{n+1}$.