

**1a** For  $n \geq 1$  we have  $a_{n+1} = (-1)^{n+1}(|a_n| + 3)$ , with  $a_1 = -4$ .

**1b**  $a_n = (-1)^n(3n + 1)$  for  $n \geq 1$

**2** Since 82% of the isotope remains at the end of each 12-year period, we have  $M_n = 80(0.82)^n$  for  $n \geq 0$

**3** We have

$$\lim_{n \rightarrow \infty} \frac{4n - 5n^6}{11n^6 - 9} = \lim_{n \rightarrow \infty} \frac{4n^{-5} - 5}{11 - 9n^{-6}} = \frac{4(0) - 5}{11 - 9(0)} = -\frac{5}{11}.$$

**4** Get this into the form  $\sum_{n=0}^{\infty} ar^n$ :

$$\sum_{n=0}^{\infty} \left(\frac{1}{6}\right)^n 3^{4-n} = \sum_{n=0}^{\infty} 81 \left(\frac{1}{18}\right)^n = \frac{81}{1 - \frac{1}{18}} = \frac{1458}{17}.$$

**5** By partial fraction decomposition,

$$\frac{1}{(n+2)(n+4)} = \frac{1}{n+2} - \frac{1}{n+4},$$

so for each  $n \geq 1$  we have

$$\begin{aligned} s_n &= \sum_{k=1}^n \frac{1}{(k+2)(k+4)} = \sum_{k=1}^n \left( \frac{1}{k+2} - \frac{1}{k+4} \right) \\ &= \left( \frac{1}{3} - \frac{1}{5} \right) + \left( \frac{1}{4} - \frac{1}{6} \right) + \left( \frac{1}{5} - \frac{1}{7} \right) + \left( \frac{1}{6} - \frac{1}{8} \right) + \cdots + \left( \frac{1}{n} - \frac{1}{n+2} \right) + \left( \frac{1}{n+1} - \frac{1}{n+3} \right) + \left( \frac{1}{n+2} - \frac{1}{n+4} \right) \\ &= \frac{1}{3} + \frac{1}{4} - \frac{1}{n+3} - \frac{1}{n+4}, \end{aligned}$$

so

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+4)} = \lim_{n \rightarrow \infty} s_n = \lim_{n \rightarrow \infty} \left( \frac{1}{3} + \frac{1}{4} - \frac{1}{n+3} - \frac{1}{n+4} \right) = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}.$$

**6a** Making the substitution  $u = \ln x$ ,

$$\int_2^{\infty} \frac{1}{x(\ln x)^2} dx = \lim_{b \rightarrow \infty} \int_2^b \frac{1}{x(\ln x)^2} dx = \lim_{b \rightarrow \infty} \int_{\ln 2}^{\ln b} \frac{1}{u^2} du = \lim_{b \rightarrow \infty} \left[ \frac{1}{\ln 2} - \frac{1}{\ln b} \right] = \frac{1}{\ln 2}.$$

So the integral converges, and therefore the series converges by the Integral Test.

**6b** Since

$$\lim_{n \rightarrow \infty} \frac{1}{n + 1000} = 0,$$

the Divergence Test is inconclusive.

**6c** Since

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{[(n+1)!]^2 \cdot (2n)!}{(2n+2)! \cdot (n!)^2} \right| = \lim_{n \rightarrow \infty} \frac{(n+1)^2}{(2n+1)(2n+2)} = \lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1}{4n^2 + 6n + 2} = \frac{1}{4} < 1,$$

the series converges by the Ratio Test.

**6d** Since

$$\lim_{n \rightarrow \infty} |a_n|^{1/n} = \lim_{n \rightarrow \infty} \left[ \left( \frac{n}{n+1} \right)^{2n^2} \right]^{1/n} = \lim_{n \rightarrow \infty} \left( \frac{n}{n+1} \right)^{2n} = e^{-2} \approx 0.1353 < 1,$$

the series converges by the Root Test.

**6e** For each  $n \geq 1$  we have

$$0 \leq \frac{1}{n^2 + 8} \leq \frac{1}{n^2},$$

and since  $\sum_{n=1}^{\infty} 1/n^2$  is a convergent  $p$ -series, it follows that

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 8}$$

converges by the Direct Comparison Test.

**6f** We could use the Limit Comparison Test:

$$\lim_{n \rightarrow \infty} \left( \frac{\sqrt[3]{n^2 + 1}}{\sqrt{n^3 + 2}} \div \frac{1}{n} \right) = \lim_{n \rightarrow \infty} \frac{n^{5/3} \sqrt{1 + 1/n^2}}{n^{3/2} \sqrt[3]{1 + 2/n^3}} = \lim_{n \rightarrow \infty} \frac{n^{1/6} \sqrt{1 + 1/n^2}}{\sqrt[3]{1 + 2/n^3}} = \infty,$$

since

$$\lim_{n \rightarrow \infty} n^{1/6} = \infty \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{\sqrt{1 + 1/n^2}}{\sqrt[3]{1 + 2/n^3}} = \frac{\sqrt{1 + 0}}{\sqrt[3]{1 + 0}} = 1.$$

Now,  $\sum \frac{1}{n}$  is a divergent  $p$ -series, and so by the Limit Comparison Test we conclude that the given series diverges.

**7a** Observe that, using L'Hôpital's Rule,

$$\lim_{n \rightarrow \infty} \frac{n}{\ln n} = \lim_{n \rightarrow \infty} \frac{1}{1/n} = \lim_{n \rightarrow \infty} n = \infty,$$

so that

$$\lim_{n \rightarrow \infty} (-1)^n \frac{n}{\ln n} \neq 0$$

and the Divergence Test implies that the series diverges.

**7b** The sequence

$$\left(\frac{\sqrt{n}}{n+6}\right)_{n=1}^{\infty}$$

is monotone decreasing, and moreover

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n+6} = \lim_{n \rightarrow \infty} \frac{n^{-1/2}}{1+6/n} = 0.$$

Therefore the series converges by the Alternating Series Test.