MATH 141
FALL 2021
EXAM 4

NAME:

1. (a) [10 pts.] Find the quadratic approximating polynomial for $f(x) = \sqrt{x}$, centered at $a = 4$.
   (b) [5 pts.] Use the quadratic approximating polynomial to approximate $\sqrt{3.88}$.

2. [10 pts.] Use the remainder term to find a bound on the absolute error of the approximation

$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

on the interval $[-0.12, 0.14]$.

3. [10 pts. each] Determine the interval of convergence of the power series.

(a) $\sum \frac{(-1)^n n^2}{(n+1)!}(x+3)^n$ (b) $\sum \frac{6^n}{\sqrt{n}} x^n$ (c) $\sum \frac{(-1)^n}{n^2 3^n}(x-2)^n$

4. [15 pts.] Find a power series representation centered at 0 for the function

$$f(x) = \ln\sqrt{1-x^2},$$

and determine the interval of convergence of the series.

5. [10 pts.] Find the first four nonzero terms of the binomial series centered at 0 for

$$f(x) = (1+2x)^{3/4}.$$

6. [10 pts.] Use Maclaurin series (see table on other side) to evaluate the limit

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x \arctan x}.$$

7. [10 pts.] Approximate the value of the definite integral with an absolute error less than 10^{-4}:

$$\int_0^1 \sin\sqrt{x}\, dx.$$

8. [10 pts.] Express the curve given by the parametric equations

$$x(t) = \sec t, \quad y(t) = \tan t, \quad 0 \le t \le \pi/4$$

by an equation in x and y (i.e. a Cartesian equation).

9. [10 pts.] Write the equation $(x-4)^2 + y^2 = 16$ in polar coordinates, and give an interval of θ values that traces the graph of the circle precisely once.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}.$$