MATH 141
FALL 2021
EXAM 3

NAME:

1. [10 pts. each] Find the limit of each sequence, or show that the limit does not exist.

(a) $\left(n \sin \frac{\pi}{n}\right)_{n=1}^{\infty}$

(b) $\left(\sqrt{n^4 - 2n} - n^2\right)_{n=2}^{\infty}$

(c) $\left(\frac{20 \tan^{-1} n}{n^2 + 8}\right)_{n=0}^{\infty}$

2. [10 pts.] Evaluate the geometric series $\sum_{n=3}^{\infty} \frac{6}{4^n}$.

3. [10 pts.] Either show the telescoping series

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+2}{n}\right)$$

diverges, or find a formula for the nth partial sum s_n and evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

4. [10 pts.] Apply a remainder theorem to estimate, using the fewest possible terms, the value of the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{10n^4}$$

with an absolute error less than 10^{-4}. Do not bother adding the terms.

5. [10 pts. each] Determine whether the series converges or diverges, using an appropriate test and justifying all work. Arguments must be clear and thorough.

(a) $\sum_{n=0}^{\infty} \frac{4}{2 + 3^n n}$

(b) $\sum_{n=1}^{\infty} \frac{4^n}{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2}$

(d) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

(e) $\sum_{n=1}^{\infty} n^{-1/n}$

(f) $1 + \frac{1 \cdot 3}{3!} + \frac{1 \cdot 3 \cdot 5}{5!} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{7!} + \cdots$

6. 10 pts. each Use the Alternating Series Test to show the series converges, or use some other test to show it diverges. If the series converges, use any test to determine whether it converges absolutely or conditionally.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{5/4}}$

(b) $\sum_{n=3}^{\infty} \frac{(-1)^n n}{\ln n}$

Some Formulas

- $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \frac{1}{1+x^2}$
- $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$
- $\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\int \tan x\, dx = \ln|\sec x| + c$
- $\int \cot x\, dx = \ln|\sin x| + c$
- $\int \sec x\, dx = \ln|\sec x + \tan x| + c$
- $\int \csc x\, dx = -\ln|\csc x + \cot x| + c$