MATH 141
FALL 2021
EXAM 2

NAME:

1. [10 pts.] Find the integral:

$$\int \frac{1}{x^{1/2} + x^{3/2}} dx.$$

2. [10 pts.] Use integration by parts to evaluate

$$\int_0^{\ln 2} xe^{-4x} dx.$$

3. [10 pts.] Use integration by parts to determine

$$\int \varphi \csc^2 \varphi \, d\varphi.$$

4. [10 pts.] Determine the trigonometric integral

$$\int \tan^3 x \sec^2 x \, dx.$$

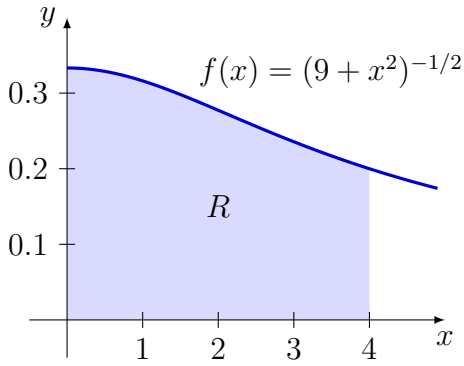
5. [10 pts.] Use a trigonometric substitution to evaluate

$$\int_6^{6\sqrt{3}} \frac{q^2}{(q^2 + 36)^2} dq.$$

6. [10 pts. each]

   (a) Find the area of the region R bounded by the curves $f(x) = (9 + x^2)^{-1/2}$, $x = 0$, $x = 4$, and $y = 0$. See figure below.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.



7. [10 pts. each] Use partial fractions to find the indefinite integral.

(a) $\displaystyle\int \frac{12}{(2s-1)(s-6)}\,ds$

(b) $\displaystyle\int \frac{x-5}{x^2(x+1)}\,dx$

8. [10 pts. each] Evaluate the improper integral, or show that it diverges.

(a) $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$

(b) $\displaystyle\int_0^1 z\ln z\,dz$

9. [10 pts.] Use the Comparison Theorem to determine whether the integral converges or diverges:

$$\int_1^\infty \frac{1+\sin^2 x}{\sqrt{x}}\,dx.$$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\Big(\frac{x}{a}\Big)+c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\Big(\frac{x}{a}\Big)+c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\Big|\frac{x}{a}\Big|+c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2}x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x|+c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x|+c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x+\tan x|+c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x+\cot x|+c$