MATH 141
FALL 2020
EXAM 3

NAME:

1. $\boxed{\text{10 pts. each}}$ Find the limit of each sequence, or show that the limit does not exist.

   (a) $\left(n \sin \dfrac{\pi}{n}\right)_{n=1}^{\infty}$

   (b) $\left(n - \sqrt{n^2 - 1}\right)_{n=1}^{\infty}$

2. $\boxed{\text{10 pts.}}$ Use the Squeeze Theorem to find the limit of the sequence

$$a_n = \frac{(3n^2 + 2n + 1)\sin n}{4n^3 + n}, \quad n \geq 1.$$

3. $\boxed{\text{10 pts.}}$ Evaluate the geometric series $\displaystyle\sum_{n=1}^{\infty} \frac{6}{4^n}$.

4. $\boxed{\text{10 pts.}}$ Either show the telescoping series

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$$

diverges, or find a formula for the nth partial sum s_n and evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

5. $\boxed{\text{10 pts.}}$ Apply a remainder theorem to estimate, using the fewest possible terms, the value of the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^4}$$

with an absolute error less than 10^{-3}. Do not bother adding the terms.

6. $\boxed{\text{10 pts. each}}$ Determine whether the series converges or diverges, using an appropriate test and justifying all work. Arguments must be clear and thorough.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{4}{2 + 3^n n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n^2}$

   (c) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n^{4/3}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} n^{-1/n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{2} - \tan^{-1} n\right)$

7. $\boxed{\text{10 pts. each}}$ Determine whether the series converges absolutely, converges conditionally, or diverges. Justify all arguments by stating what tests are being used and showing work.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{5/4}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}+6}$

Some Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$