MATH 141
FALL 2020 NAME:
EXAM 2

1. [10 pts. each] Evaluate the integral by integrating by parts.

   (a) $\int t \cos 8t \, dt$

   (b) $\int x^2 \ln^2 x \, dx$

2. [10 pts. each] Evaluate the trigonometric integral.

   (a) $\int \sin^5 6t \cos^2 6t \, dt$

   (b) $\int \tan^3 3\theta \sec^3 3\theta \, d\theta$

3. [10 pts. each] Evaluate using trigonometric substitution.

   (a) $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} \, dx$

   (b) $\int \frac{dx}{(1+x^2)^{3/2}}$

4. [10 pts. each] Evaluate using partial fractions.

   (a) $\int_1^2 \frac{7x-2}{3x^2-2x} \, dx$

   (b) $\int \frac{x^4+1}{x^3+9x} \, dx$

5. [10 pts. each] Evaluate using any strategy.

   (a) $\int \frac{1}{e^x \sqrt{1+e^{2x}}} \, dx$

   (b) $\int s^2 \tan^{-1} s \, ds$

6. [10 pts. each] Determine whether the integral is convergent or divergent, and evaluate if convergent.

   (a) $\int_{-\infty}^{-1} \frac{dx}{\sqrt[3]{x}}$

   (b) $\int_0^{\infty} \frac{t^3}{1+t^8} \, dt$

(c) $\displaystyle\int_1^{10} \frac{dx}{(x-2)^{4/3}}$

7. [10 pts.] Find the volume of the solid of revolution generated when the region bounded by $f(x) = x^{-2}$ and the x-axis on the interval $[1, \infty)$ is revolved about the x-axis.

8. [10 pts.] Use the Comparison Theorem to determine whether the integral converges or diverges:

$$\int_3^{\infty} \frac{x^3}{\sqrt{x^7 - 1}}\, dx.$$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\, dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\, dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\, dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\, dx = -\ln|\csc x + \cot x| + c$
- $\displaystyle\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx$
- $\displaystyle\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx$
- $\displaystyle\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\, dx$
- $\displaystyle\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\, dx$