MATH 141
FALL 2020
EXAM 1

NAME:

1. [10 pts.] Given $f(x) = \sqrt{x^3 + 4x + 4}$, find $(f^{-1})'(3)$ using the Inverse Function Theorem.

2. [10 pts. each] Find the derivative.

(a) $\dfrac{d}{dx}\left[\ln\left(\dfrac{e^x}{2+e^x}\right)\right]$

(b) $\dfrac{d}{dt}(1-3t)^{2t}$

(c) $\dfrac{d}{dr}[r \log_6(1/r)]$

(d) $\dfrac{d}{dy}(\tan^{-1} y^2 - \cos^{-1}\sqrt{y})$

(e) $\dfrac{d}{dx}\tanh^3(e^{2x})$

3. [10 pts.] Find an equation of the tangent line to $y = 2e^x - 1$ at $x = \ln 3$.

4. [10 pts. each] Evaluate each integral.

(a) $\displaystyle\int_0^3 \frac{2p-1}{p+1}\,dp$

(b) $\displaystyle\int \frac{dy}{y\sqrt{4y^2-100}}$

(c) $\displaystyle\int 4^{-2x}\,dx$

(d) $\displaystyle\int_0^{\ln 4} \frac{e^x}{3+2e^x}\,dx$

5. [10 pts.] Find the critical points of $f(x) = xe^{-x^2}$, and use the First Derivative Test to locate the local maximum and minimum values.

6. [10 pts. each] Evaluate the limit using L'Hôpital's Rule, or state the limit does not exist.

(a) $\displaystyle\lim_{x\to 0^+}(e^{3x}+x)^{1/x}$

(b) $\displaystyle\lim_{t\to 1^+}\left(\sqrt{t}-1\right)^{\sin \pi t}$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \dfrac{1}{1+x^2}$
- $(\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c$
- $\displaystyle\int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c$
- $\displaystyle\int \csc x\,dx = -\ln|\csc x + \cot x| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$