MATH 141
FALL 2019
EXAM 4

NAME:

1. [10 pts.] Approximate $\sqrt{9.3}$ with a 3rd-order Taylor polynomial having an appropriate center.

2. [10 pts. each] Determine the interval of convergence of the power series.

(a) $\sum \frac{2^{2n}x^n}{n^2}$ (b) $\sum \frac{(x+1)^n}{n \cdot 6^n}$ (c) $\sum n!(x-3)^n$

3. [15 pts.] Find a power series representation for the function

$$f(x) = \frac{2x}{1-x^4},$$

and determine the interval of convergence.

4. [15 pts.] Find a power series representation for the function $f(x) = \sqrt{1+x}$, determine the interval of convergence, and write out the first four nonzero terms of the series.

5. [10 pts.] Use Taylor series to evaluate the limit

$$\lim_{x \to 0} \frac{1-\cos x}{1+x-e^x}.$$

6. [10 pts.] Use a Taylor series to approximate the value of the definite integral

$$\int_0^{1/3} e^{-x^2}\, dx$$

with an absolute error less than 10^{-4}.

7. [10 pts. each] Consider the parametric equations

$$x = \sqrt[3]{t^8 - 8}, \quad y = \sqrt{t^4 + 1}$$

for $-\infty < t < \infty$.

(a) What is the slope of the curve at the point corresponding to $t = 0$?

(b) Eliminate the parameter to obtain an equation in x and y.

8. [10 pts.] An object moves along a straight path from the point $(-2, 5)$ at time $t = 0$ to the point $(2, -1)$ at time $t = 10$. Find a parametric description of the object's path.

9. [10 pts.] Convert the polar equation to Cartesian coordinates:

$$r = \frac{2}{4\cos\theta + 3\sin\theta}.$$

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \leq b_{k+1} \leq b_k$ for all k, then $R_n \leq b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \leq 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \leq 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}.$$